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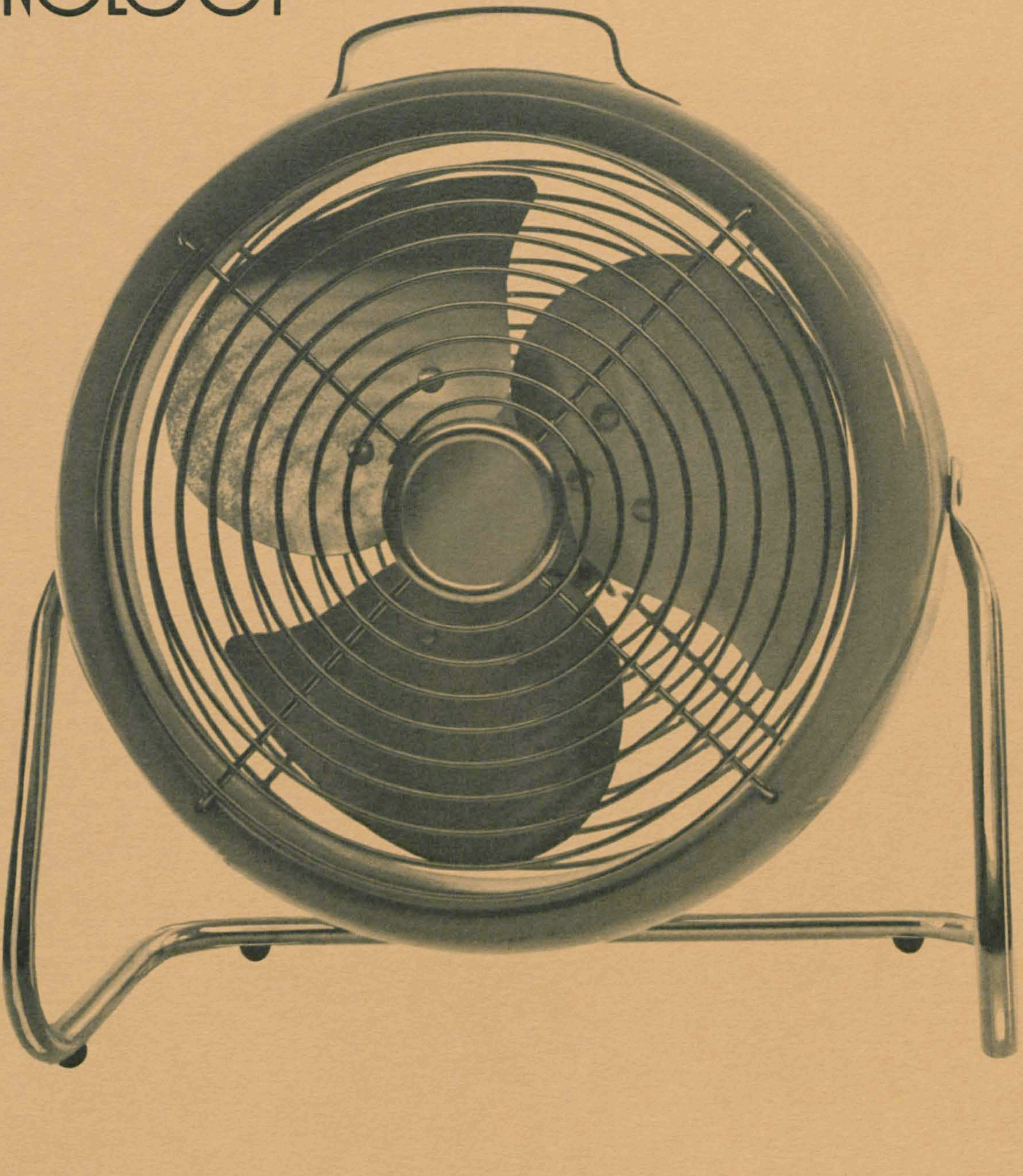

THE ELECTRIC FAN

Rigid Body Rotation

THE ELECTRIC FAN

A Module on Rigid Body Rotation

TERC

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The Electric Fan

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PREFACE

ABOUT THIS MODULE

Its Purpose

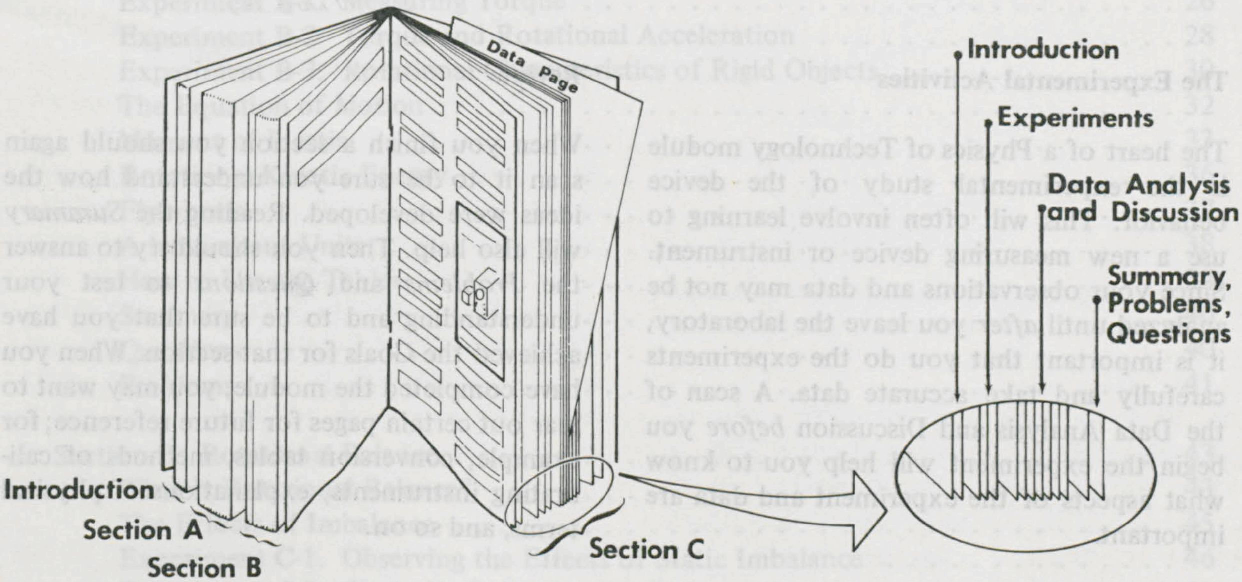
The purpose of the Physics of Technology program is to give you an insight into the physical principles that are the basis of technology. To do this you are asked to study various technological devices. These devices have been chosen because their operation depends on or illustrates some important physical phenomena. In this module the device is the electric fan. Its design and use involve rotational motion.

The PoT program has adopted a modular format with each module focusing on a single device. Thus you can select only those modules that relate to your own interests or areas of specialty. This preface highlights some of the features of the modular approach so that you may use it efficiently and effectively.

Its Design

The *Introduction* explains why we have selected the electric fan to study and what physical principles will be illustrated in its behavior. There are learning *Goals* and a self-test for *Prerequisite* skills you should have before beginning. The three *Sections* of the module treat different aspects of the electric fan. They are of increasing difficulty but each can be completed in about a week.

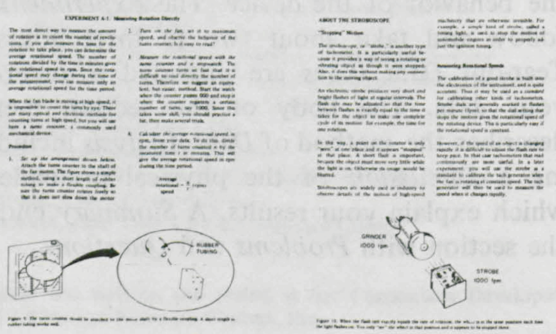
Each section begins with a brief *Introduction* to the topics treated and how they relate to the behavior of the device. The *Experiments* follow and take about two to three hours. Tear-out *Data Pages* are provided to record your data. The body of the section then describes the method of *Data Analysis* including a *Discussion* of the physical principles which explain your results. A *Summary* ends the section with *Problems* and *Questions.*



HOW TO USE THIS MODULE

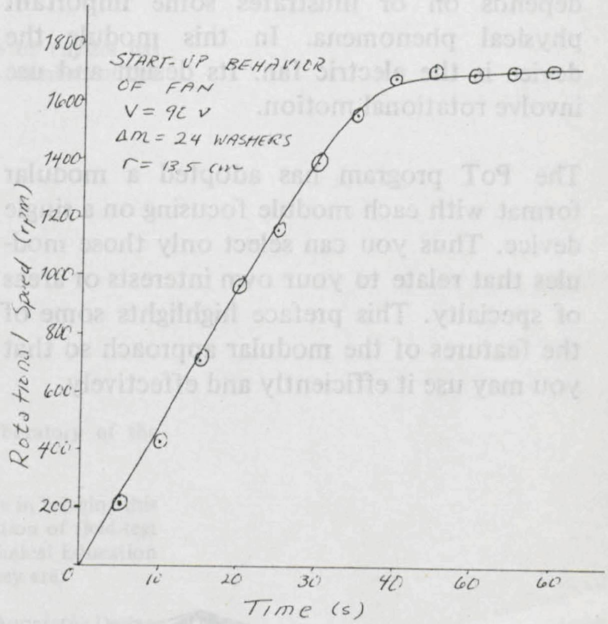
To Begin

This module has been written so that it can be quickly and easily scanned. That is, you can get the gist of the ideas and experiments by simply flipping from page to page, reading only the headings and italicized words, and looking at the illustrations. We suggest that before you begin a section or an experiment, you scan through it in this way so that you will know where you are going.



The Data Analysis

The data you take will generally have to be graphed before they can be analyzed. Graphing and graphical analysis are essential parts of experimental science. Understanding graphs also is important for technology, since technical information is often presented graphically. For these reasons, and since the discussion of your results will be centered around your graphs, it is important that you prepare them clearly and accurately.



The Experimental Activities

The heart of a Physics of Technology module is the experimental study of the device behavior. This will often involve learning to use a new measuring device or instrument. Since your observations and data may not be analyzed until *after* you leave the laboratory, it is important that you do the experiments carefully and take accurate data. A scan of the Data Analysis and Discussion *before* you begin the experiment will help you to know what aspects of the experiment and data are important.

Review

When you finish a section you should again scan it to be sure you understand how the ideas were developed. Reading the *Summary* will also help. Then you should try to answer the *Problems* and *Questions* to test your understanding and to be sure that you have achieved the Goals for that section. When you have completed the module, you may want to tear out certain pages for future reference; for example, conversion tables, methods of calibrating instruments, explanations of physical terms, and so on.

TABLE OF CONTENTS

The Electric Fan

INTRODUCTION. WHY STUDY AN ELECTRIC FAN?

Fans Use Rotational Motion

In a study of an electric fan, you might expect to learn about the behavior of air and of air flow. While such behavior involves some important physics ideas, in this module you will study another important aspect of fans, *rotational motion.* In order for the fan to work, the fan blade has to rotate at high speed. The behavior of objects that rotate at high speed is of considerable importance in technology as well as in daily life.

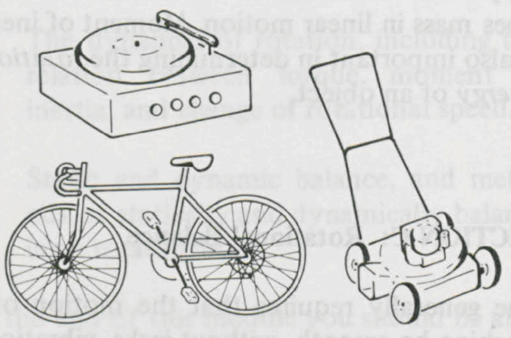


Figure 1. Common devices that use rotational motion.

Rotational Motion Is Common in Technology

One of the fundamental technological advances in history, the invention of the wheel, depends on rotational motion. Today one can make the statement that nearly every technological device that moves involves rotational motion somewhere! How many exceptions to that claim can you think of? Not many. The illustration above shows a few of many possible examples of devices that use rotational motion.

While we might have designed a module around any one of these devices, the electric fan has a number of advantages for a study of the behavior of rotating objects. It is inexpensive, its rotational characteristics are easily changed and measured, and it has a protective grill.

The Principles of Rotation Have Many Applications

Probably the most common example to which the principles of rotation are relevant is the automobile. The speedometer really measures the *rotational speed* of a wheel. The wheels as well as the crankshaft must be properly *balanced,* both *statically* and *dynamically*, so that potentially dangerous vibrations do not occur. The motion of the car itself depends on the *torque* developed by the engine.

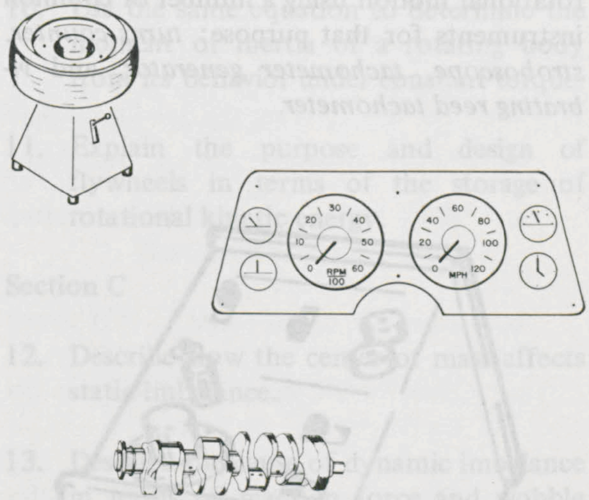


Figure 2. The principles of rotational motion have many applications in automobiles.

Another common application of rotational motion is the flywheel of a small engine, like those in lawn mowers. The flywheel stores *rotational energy* from one power stroke to the next to maintain a steady rotational speed.

WHAT WILL YOU LEARN?

SECTION A: Describing Rotational Motion

Before you begin to understand rotational motion, you must learn to describe it accurately. The description of motion is called *kinematics,* and you will be studying *rotational kinematics.* The terms used to describe straight-line motion (linear kinematics) are "distance traveled," "speed," and "acceleration." The corresponding terms in rotational kinematics are *angle of rotation, rotational speed,* and *rotational acceleration.*

In Section A you will learn about these terms and about the units used to measure them. Equally important, you will learn how to relate these rotational quantities to various aspects of linear motion.

In your experimental study of the motion of a fan you will also learn how to measure the rotational motion using a number of common instruments for that purpose: *turns counter, stroboscope, tachometer generator,* and *vibrating reed tachometer.*

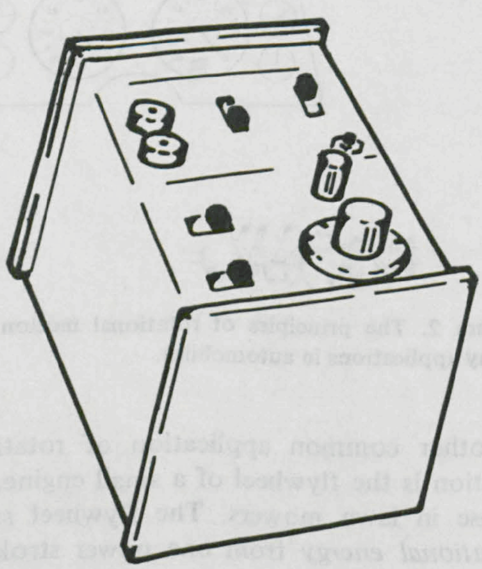


Figure 3. The stroboscope is an important instrument for measuring and observing rotational motion.

SECTION B: Explaining Rotational Motion

Rotational motion is rarely constant. At various times it speeds up or slows down in response to outside influences. These outside influences are called *torques,* and they affect rotational motion in much the same way as forces affect motion along a line. The study of what causes a particular type of motion is called dynamics. Thus the topic of Section B is *rotational dynamics.*

In your experiments you will measure the way in which the fan speeds up and slows down, and how the behavior is different for different rotating objects. Not only is the mass of the object important, but so is its shape. The relevant characteristic of a rotating body is called its *moment of inertia,* which plays the same role in rotational motion as does mass in linear motion. Moment of inertia is also important in determining the *rotational energy* of an object.

SECTION C: Rotational Balance

One generally requires that the motion of a machine be smooth, without jerks, vibrations, or wobbles. For rotational motion, this usually means that the rotating object must be properly *balanced.*

Two types of balance are important in rotational motion, *static balance* and *dynamic balance.* An object is in static balance when it has no tendency to turn on its axis when stationary in any position. An object is in dynamic balance when it has no tendency to wobble when rotating. Imbalances of either type can produce vibrations which may be annoying or even dangerous.

In Section C you will observe the characteristic behavior of objects that are either not in static or not in dynamic balance. You will also learn what causes both types of unbalanced conditions and how to cure them.

GOALS

The general goal of this module is to give you an understanding of the important principles of rotational motion and how these principles may be applied to real rotating objects.

This understanding involves a knowledge of:

The terms used to describe rotational motion (angle turned, rotational speed, rotational acceleration) and their relations to corresponding terms for linear motion.

The instruments used to observe and measure rotational motion, such as tachometers, revolution counters, and stroboscopes.

The dynamics of rotation, including the relation between torque, moment of inertia, and change of rotational speed.

Static and dynamic balance, and methods of statically and dynamically balancing a rotating body.

At the end of this module you should be able to demonstrate your understanding by doing the following things.

Section A

1. Describe how to use a stroboscope to measure rotational speed.

2. Use a stroboscope to make a calibration graph for a tachometer generator.

3. Measure the rotational speed of a rotating object as that speed changes with time.

4. Use, and convert between, the common units of rotational measure: degrees and radians.

5. Determine the rotational acceleration of a rotating object from a graph of rotational speed versus time.

6. Calculate the linear distance traveled by, and the linear velocity of, a point on a rotating body.

Section B

7. Measure and explain the stall torque of a motor.

8. Explain the moment of inertia I and be able to compute it for bodies of various simple shapes.

9. Use the basic rotational equation

$$\tau = I\alpha$$

to analyze changes in rotational motion.

10. Use the same equation to determine the moment of inertia of a rotating body from its behavior under constant torque.

11. Explain the purpose and design of flywheels in terms of the storage of rotational kinetic energy.

Section C

12. Describe how the center of mass affects static imbalance.

13. Describe the cause of dynamic imbalance in terms of reaction force and wobble torque.

14. Statically balance a rotor using a simple balancing machine.

15. Dynamically balance a rotor using a trial and error technique.

16. Explain how an unbalanced rotor produces vibrations, and suggest three ways to reduce the vibrations.

PREREQUISITES

Before you begin this module, you will need some basic knowledge of a very few physical quantities. The following self-test will tell you whether you have satisfied the prerequisites. If you can answer all of the questions, you are ready to begin. If you have difficulties with any question, get some help from your teacher or another student.

1. A car travels from Cleveland, Ohio to Buffalo, New York, a distance of about 300 km (190 mi), in four hours. What was its average speed?

2. A dragster accelerates uniformly from a standstill to 40 m/s (90 mph) in 5 s.

   a. What was its average speed during that period?

   b. How far did it travel?

   c. What was its acceleration?

3. The net force acting on a 1-kg mass is 10 N.

   a. What is the speed of the mass after 10 s, if it starts from rest?

   b. How far did it travel in that time?

   c. How much work did the force do on it?

   d. What was its kinetic energy at the end of the 10 s?

Answers to the Prerequisites Test

1. $s_{av} = \dfrac{d}{t} = \dfrac{300\ \text{km}}{4\ \text{h}} = 75\ \text{km/h}$

2. a. $s_{av} = \dfrac{s_{final} + 0}{2} = \dfrac{40\ \text{m/s}}{2}$

   $= 20\ \text{m/s}$

   b. $d = s_{av}t = (20\ \text{m/s})(5\ \text{s})$

   $= 100\ \text{m}$

   c. $a = \dfrac{s_{final} - 0}{t} = \dfrac{40\ \text{m/s}}{5\ \text{s}}$

   $= 8\ \text{m/s}^2$

3. a. $F = ma$

   $a = \dfrac{F}{m} = \dfrac{10\ \text{N}}{1\ \text{kg}} = 10\ \text{m/s}^2$

   $s_{final} = at = (10\ \text{m/s}^2)(10\ \text{s})$

   $= 100\ \text{m/s}$

   b. $d = s_{av}t = \dfrac{s_{final}}{2}t$

   $= (50\ \text{m/s})(10\ \text{s}) = 500\ \text{m}$

   c. $W = Fd = (10\ \text{N})(500\ \text{m})$

   $= 5000\ \text{J}$

   d. $KE = ½mv^2 = ½(1\ \text{kg})(100\ \text{m/s})^2$

   $= 5000\ \text{J}$

SECTION A

Describing Rotational Motion

HOW IS ROTATIONAL MOTION DESCRIBED?

Suppose you are driving on a flat road and someone asks you to "describe your motion" at a particular instant of time. How do you answer? Take a moment and try to list as many aspects of the motion as you can, giving as complete a description as possible. To help in your thinking, assume that the car has a compass in it.

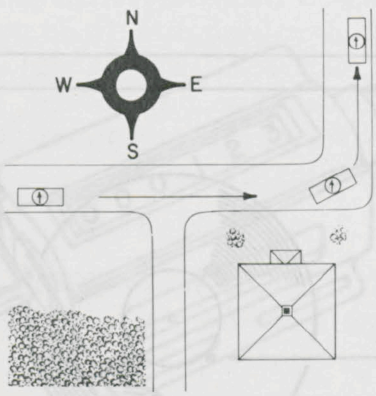


Figure 4. How would you describe the motion of a car?

Probably the first thing on your list is your speed in miles per hour. But you would also have to include such things as whether you are accelerating or decelerating, which direction you are going (north, south, etc.), whether that direction is changing, and if so, how fast.

If you look at the list, you may see that it can be divided into two categories. Those items that describe the changing position of the car refer to its *translational* motion. Those that describe the turning motion of the car with respect to the compass needle refer to its *rotational* motion. Thus the motion of the car can be separated into translational and rotational parts.

The Motion of Rigid Objects

It is generally true for any rigid object that the motion, no matter how complicated, can be separated into a translational motion and a rotational motion. Therefore, if you have previously studied translational motion, you will now be studying the only other kind of motion that a rigid object can have.

You may well ask, "Why specify a *rigid* object?" A rigid object keeps its shape as it moves. Such things as a rope, a spring, or a plastic bag filled with water do not. These objects also can have other kinds of motion (vibrations, flow, etc.) which, though often important, are not treated in this module.

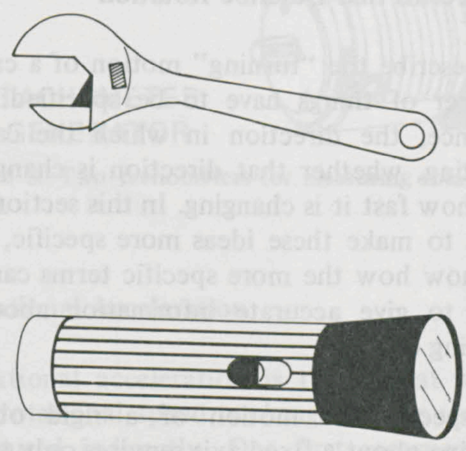


Figure 5. We are concerned here with rigid objects.

Rotation on a Fixed Axis

You will study in this module rigid objects that rotate about a *fixed axis.* This particular type of rotational motion is very frequently encountered in technology. Such motion is found in almost anything that has a wheel or a motor: automobile wheels, crankshafts, propellor blades, motor armatures, etc.

In most cases of interest, the axis of rotation is an *axle* of some sort; for example, the axle of an automobile wheel. Saying that this axle is "fixed" means that its direction in space, like that of a compass needle, does not change. The axle can move, such as when the car moves along a straight road. But if it turns, such as when the car goes around a corner, the true axis of rotation is no longer exactly the axle. The description of the rotational motion then becomes more complicated.

However, if the turning is slow relative to the rotation about the axle (as is often the case for an automobile wheel), the true axis of rotation is so nearly the axle that the difference can often be neglected. Can you think of a rotating object for which the axis of rotation changes relatively rapidly?

The Terms that Describe Rotation

To describe the "turning" motion of a car, a number of things have to be specified: for instance, the direction in which the car is pointing, whether that direction is changing, and how fast it is changing. In this section we want to make these ideas more specific, and to show how the more specific terms can be used to give accurate information about a rotating object.

To specify the motion of a rigid object rotating about a fixed axis requires only three quantities: the *angle of rotation,* the *rotational speed,* and the *rotational acceleration.* Some of these quantities may be quite familiar to you, though perhaps by other names.

A Familiar Example

A phonograph turntable provides a familiar example of rotational motion. It usually has three "speeds": 33-1/3, 45, and 78. These are *rotational speeds.* The numbers refer to the number of revolutions per minute (rpm) a record makes.

When you turn on the record player, the turntable gradually speeds up to the rotational speed you have selected. That is, its rotational speed changes from 0 rpm to, say, 45 rpm. The term which describes how fast the rotational speed changes is *rotational acceleration.*

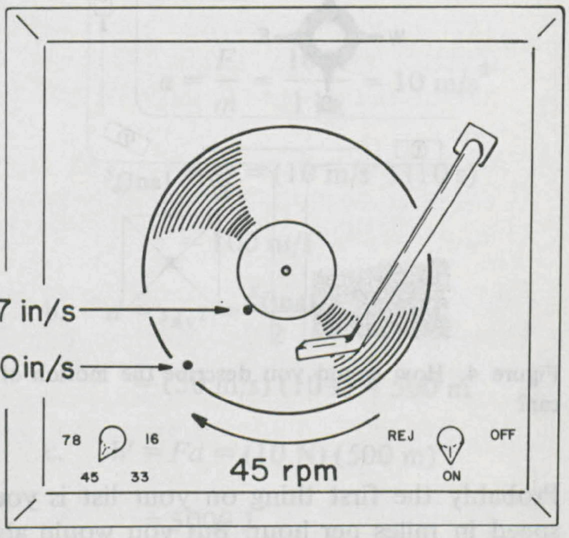


Figure 6. The record moves at a single rotational speed, but the speed of a groove beneath the needle depends on its distance from the center.

If the needle is in an outer groove, many more inches of groove pass by it in one revolution than when it is in a groove near the center. That means that the speed of the record passing beneath the needle is greater near the outer edge than near the center. In Section A you will learn to relate linear speed to the rotational speed.

HOW IS ROTATIONAL MOTION MEASURED?

Rotation Angle

The total angle of rotation through which a rotating object turns may be found from the *number of rotations* (including fractions of a rotation) that the object has made during a period of time. A simple way of measuring the angle of rotation is to use a *turns counter.*

Figure 7 shows a typical turns counter like the one you will use in your experiments. It is essentially the same as the odometer in your car, which records the number of miles the car has traveled.

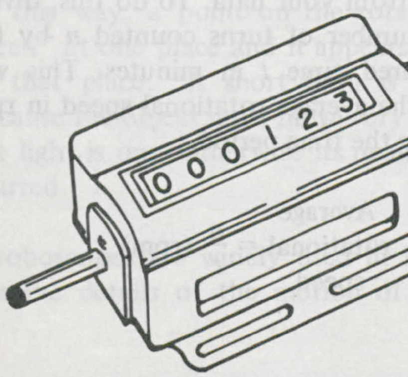


Figure 7. Turns counter for counting rotations.

Rotational Speed

Rotational speed is the rate at which an object turns. Instruments used to measure rotational speed are called *tachometers,* or frequently "tachs" for short. There are many different principles of tachometer operation. In automobiles, for example, tachometers measure the rotational speed of the engine electronically from the electrical pulses of the ignition system. In Experiment A-1 you will explore two other types of tachometer, the vibrating reed tachometer and the tachometer generator.

Vibration tachometers have metal strips, called *reeds.* Each strip vibrates strongly at a particular rate of rotation of the object. Such tachs are often used for extremely accurate measurements over a very narrow range. For example, portable electric generators must be operated at just the right rotational speed to give an AC (alternating current) voltage of 60 cycles per second. A vibration tach is usually built into such generators.

A *tachometer generator* is essentially a DC (direct current) electric generator. In a DC generator a coil of wire wound on an iron core rotates in the magnetic field of a magnet. As the coil turns, the voltage produced in the wire coil is proportional to its speed of rotation. If the generator is attached to a rotating device, its output can be used to measure the rotational speed of the device.

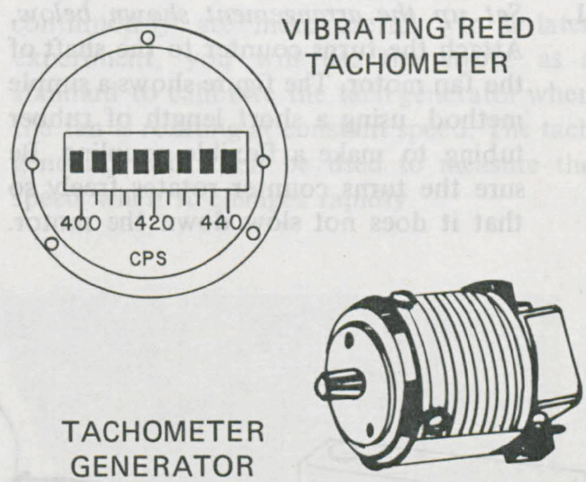


Figure 8. Two tachometers for measuring rotational speed.

Rotational Acceleration

Rotational acceleration is the rate at which the rotational speed is *changing.* It is usually measured indirectly. One first measures the rotational speed at different times using a tachometer. Then, from a graph of this time behavior, one can calculate the rotational acceleration at any specific time.

EXPERIMENT A-1. Measuring Rotation Directly

The most direct way to measure the amount of rotation is to count the number of revolutions. If you also measure the time for the rotation to take place, you can determine the average rotational speed. The number of rotations divided by the time in minutes gives the rotational speed in rpm. Since the rotational speed may change during the time of the measurement, you can measure only an *average* rotational speed for the time period.

When the fan blade is moving at high speed, it is impossible to count the turns by eye. There are many optical and electronic methods for counting turns at high speed, but you will use here a *turns counter,* a commonly used mechanical device.

Procedure

1. *Set up the arrangement shown below.* Attach the turns counter to the shaft of the fan motor. The figure shows a simple method, using a short length of rubber tubing to make a flexible coupling. Be sure the turns counter rotates freely so that it does not slow down the motor.

2. *Turn on the fan,* set it to maximum speed, and observe the behavior of the turns counter. Is it easy to read?

3. *Measure the rotational speed with the turns counter and a stopwatch.* The turns counter turns so fast that it is difficult to read directly the number of turns. Therefore we suggest an equivalent, but easier, method. Start the watch when the counter passes 000 and stop it when the counter registers a certain number of turns, say 1000. Since this takes some skill, you should practice a bit, then make several trials.

4. *Calculate the average rotational speed,* in rpm, from your data. To do this, divide the number of turns counted n by the measured time t in minutes. This will give the average rotational speed in rpm during the time period.

$$\text{Average rotational speed} = \frac{n}{t} \text{ (rpm)}$$

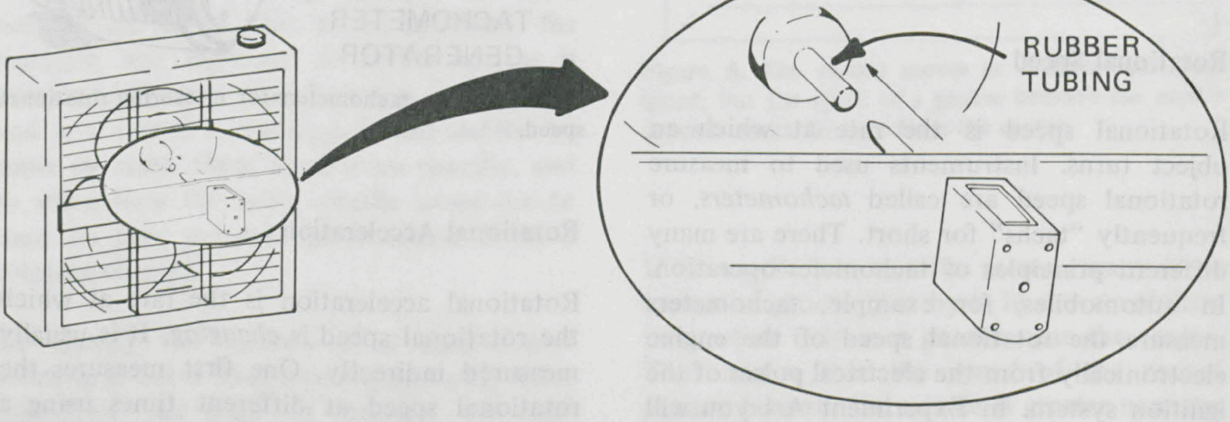


Figure 9. The turns counter should be attached to the motor shaft by a flexible coupling. A short length of rubber tubing works well.

ABOUT THE STROBOSCOPE

"Stopping" Motion

The *stroboscope,* or "strobe," is another type of tachometer. It is particularly useful because it provides a way of seeing a rotating or vibrating object as though it were stopped. Also, it does this without making any connection to the moving object.

An electronic strobe produces very short and bright flashes of light at regular intervals. The flash rate may be adjusted so that the time between flashes is exactly equal to the time it takes for the object to make one complete cycle of its motion–for example, the time for one rotation.

In this way, a point on the rotator is only "seen" at one place and it appears "stopped" at that place. A short flash is important, because the object must move very little while the light is on. Otherwise its image would be blurred.

Stroboscopes are widely used in industry to observe details of the motion of high-speed machinery that are otherwise invisible. For example, a simple kind of strobe, called a timing light, is used to stop the motion of automobile engines in order to properly adjust the ignition timing.

Measuring Rotational Speeds

The calibration of a good strobe is built into the electronics of the instrument and is quite accurate. Thus it may be used as a *standard* for calibrating other kinds of tachometers. Strobe dials are generally marked in flashes per minute (fpm), so that the dial setting that stops the motion gives the rotational speed of the rotating device. This is particularly easy if the rotational speed is constant.

However, if the speed of an object is changing rapidly it is difficult to adjust the flash rate to keep pace. In that case tachometers that read continuously are more useful. In a later experiment, you will use the strobe as a standard to calibrate the tach generator when the fan is rotating at constant speed. The tach generator will then be used to measure the speed when it changes rapidly.

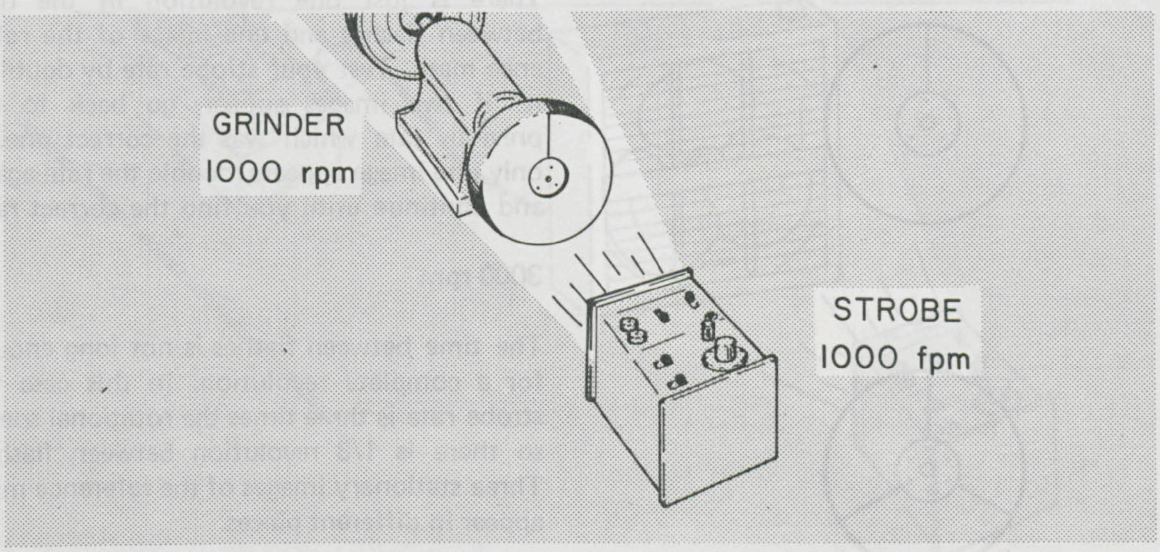


Figure 10. When the flash rate exactly equals the rate of rotation, the wheel is in the same position each time the light flashes on. You only "see" the wheel in that position and it appears to be stopped there.

How to Use a Strobe

In using a stroboscope, the first step is to make an easily visible reference mark on the rotating system. A scratch, a paint spot, or any other similar mark will serve, but it is essential that it be the *only mark of its kind.*

When the system is rotating, the goal is to change the strobe rate until that mark appears to be stationary. It is worth noting that this stopped image may appear so real that you may actually believe it is not rotating. However, *do not stick your finger in the system to see if it is rotating.* Instead change the strobe rate to see if the mark moves.

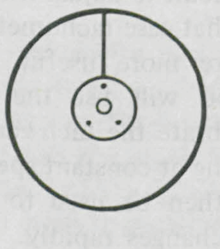
More than one strobe rate will stop the motion. Rates that are certain fractions of the real rate stop the motion, as do multiples of the real rate. Can you figure out why? It is important to remember the following rule: *The true rotational rate is the highest strobe rate that stops the motion of the reference mark in one position only.*

The illustration below shows what a rotating disk might look like for various strobe flash rates that stop the motion.

OBSERVATION

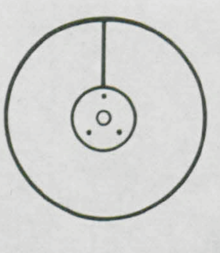
(rotational speed = 1000 rpm)

STROBE RATE



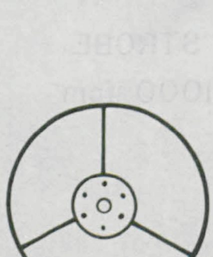
500 rpm

The motion is stopped, but the strobe rate is only half the rotational speed. Thus the rotator makes two revolutions between flashes. However, you still see the motion as stopped.



1000 rpm

There is just one revolution in the time between flashes and one image of the reference mark. Test your strobe rate by doubling it: if two images appear, go back to the previous rate which was the correct one. If only one image appears, double the rate again, and continue until you find the correct rate.



3000 rpm

The time between flashes is not long enough for a complete revolution. In this case the strobe rate is three times the rotational speed, so there is 1/3 revolution between flashes. Three stationary images of the reference mark appear in different places.

Figure 11.

EXPERIMENT A-2. Using the Stroboscope

The purpose of this experiment is to give you experience in the use of a stroboscope. You should explore its use to your own satisfaction. However, the procedure suggests a few things to do to see how the strobe can be used to observe and measure the behavior of a rotating rigid object.

Procedure

1. *Set up* the arrangement shown in the figure below.

   The fan should be plugged into the variable voltage supply and its switch turned to HIGH. Make an easily visible reference mark on one of the fan blades. Be sure the fan blade is firmly mounted to the motor's axle.

   The variable voltage supply should be plugged into a 110-V outlet. Changing the voltage supplied to the fan changes its rotational speed.

   The strobe should be positioned so it will shine on the blades. The room light should be lowered so that the strobe flashes show up brightly.

2. *Turn on the voltage supply* and increase the voltage to a maximum, so that the fan turns at top speed.

3. *Turn on the strobe* and set it to read about 1000 rpm. It may take a few seconds for the strobe to begin flashing. *Increase the flash rate slowly,* while watching the fan, until your reference mark is stopped by the flashing strobe.

4. *Explore the behavior* of the reference mark when you pass through the exact strobe rate that stops the motion. Which way does the mark move when the rate is slightly too high? Too low? Vary the rate to double, half, etc. of the true rate.

5. *Decrease the voltage* on the variable supply and try to follow the changing speed of the fan blade by changing the strobe rate. How low a rotational speed can you measure? You may have to change the range on the strobe as the speed of the fan decreases.

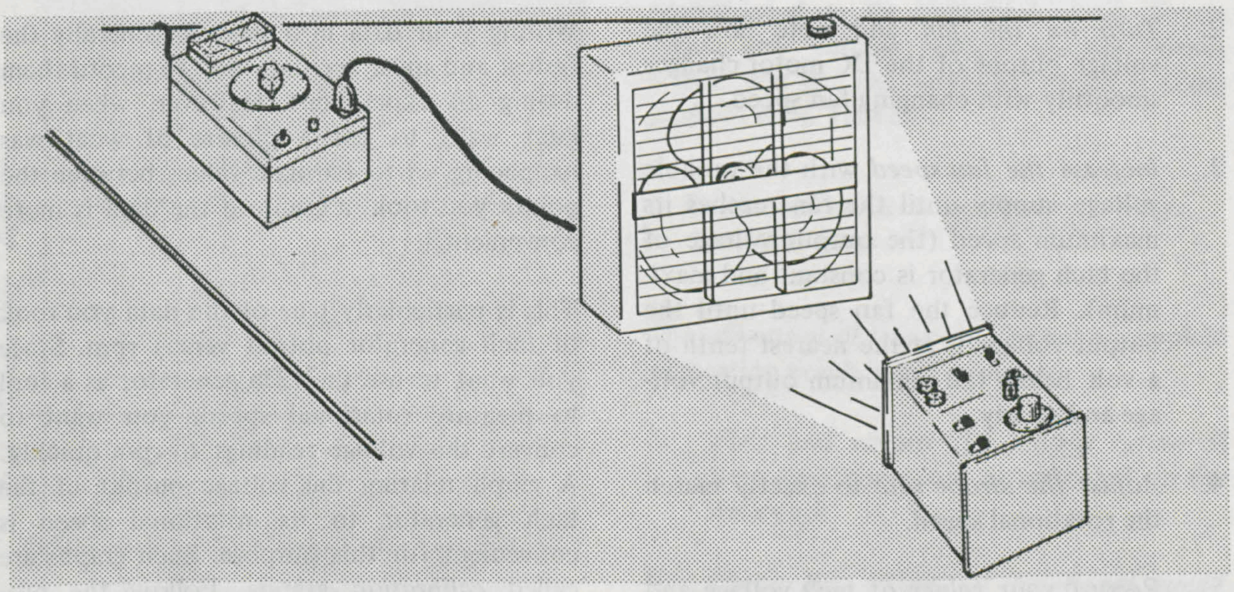


Figure 12.

EXPERIMENT A-3. Calibrating a Tach Generator

In this experiment you will calibrate a DC motor so it can be used as a tach generator. The motor is coupled to the rotating fan shaft so that it is driven as a DC generator. The voltage output of the generator can be compared to the reading of the calibrated strobe which is set to stop the motion of the rotating fan.

Several readings of tach voltage versus rotational speed should be made covering the full range of fan speeds. Later these data will be used to make a calibration graph.

Procedure

1. *Set up* the arrangement shown in Figure 13.

   The DC motor should be attached to the fan shaft with a short length of rubber tubing, as before. Be sure the coupling is firmly seated and does not slip.

   The meter should be a DC voltmeter, with a range of 0 to 1 volt. Connect the meter to the output terminals of the DC motor.

2. *Turn on the fan* and note how the voltage output of the DC motor changes smoothly with changing fan speed.

3. *Increase the fan speed* with the variable voltage supply until the fan reaches its maximum speed (the output voltage of the tach generator is constant and maximum). Reduce the fan speed until the output voltage is at the nearest tenth of a volt below the maximum output voltage and steady.

4. *Adjust the strobe rate* to exactly match the rotational speed.

5. *Record your values* of tach voltage and strobe rate in the space provided on the data page.

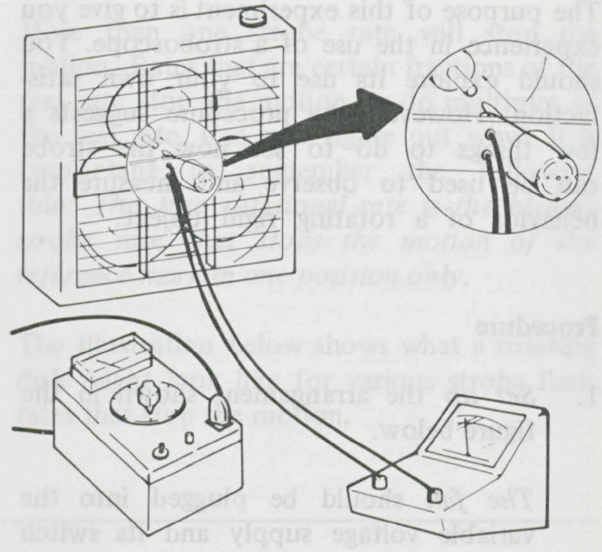


Figure 13. A DC motor acts as a tach generator.

6. *Repeat the measurement,* lowering the output of the tach generator in steps of one-tenth of a volt down to as low a speed as you can.

The Calibration Graph

Writing your data in a table is generally the fastest and most accurate way to record them during an experiment. However, when you later want to get a picture of what was happening, or to estimate values between the points you took, a graph of the data is more convenient.

This is particularly true of the data you took of tach generator output versus rpm. Since you want to use the tach generator as a tool to measure rotational speed, you want to convert the voltage readings to rpm quickly. A graph relating the voltage output of the tach generator to its rotational speed is convenient for this purpose. Such graphs are called *calibration* graphs. Follow the procedure on the next page to get a calibration graph for your tach generator.

Drawing the Calibration Graph

Normally one plots the quantity that is most easily changed on the horizontal axis of a graph. For the fan this is the rotational speed. The tachometer voltage is a reading that depends on the rotational speed setting, so it is plotted on the vertical axis.

1. *Select convenient scales* for each axis so that they are easy to read (for example, 10 divisions equals 100 rpm) and so that the range of your data fills the paper as nearly as possible. See Figure 14.

2. *Label your graph* so that you can recall exactly what it represents. Include the two variables, how they were measured, what specific apparatus was used, when the experiments were done, and by whom.

3. *Plot the points.* Put a small dot on the graph for each pair of values from your table. The dots can be made more visible by putting a geometrical shape, such as a circle, around them. Be sure this shape is centered on the dot.

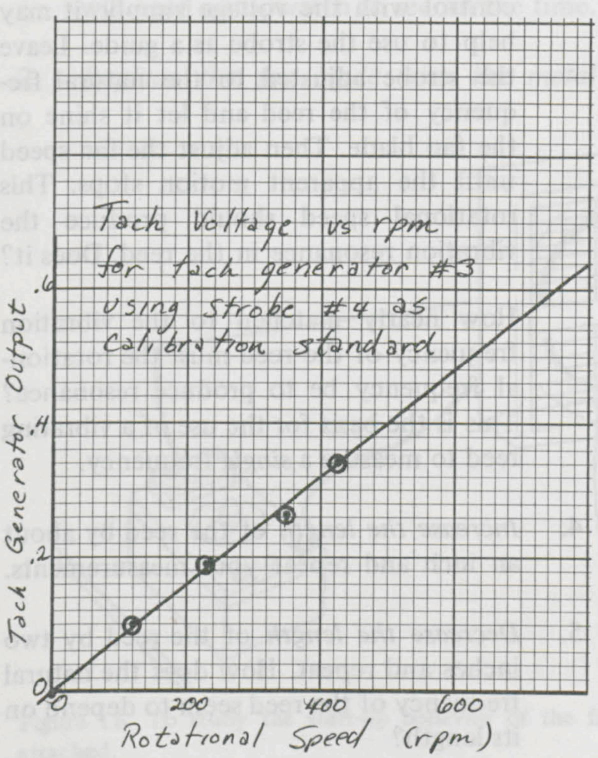


Figure 14. Typical scales to use when making your calibration graph. The graph should be clearly titled.

4. *Draw a smooth line* through the points. The points you measured should be close to being on a straight line. Therefore draw a straight line through them that intersects as many points as possible. When doing this, try to keep the number of points above the line about equal to the number below.

   Be careful when drawing this graph. It is a calibration graph, and all your measurements after this will depend on your work here. A clear plastic straightedge will help you draw your line accurately.

Calculating the Output Rating

One important specification of a tachometer generator is the number of volts it puts out for a given rotational speed. This so-called "output rating" R is generally given as so many *volts per 1000 rpm.* This can be found directly from your graph. The straight-line graph shows that the tach generator *voltage is proportional to its rotational speed.* If you divide any voltage output by the output rating, you get the rotational speed in thousands of rpm. That is:

$$\begin{array}{c}\text{Rotational speed} \\ \text{(in 1000 rpm)}\end{array} = \frac{\text{tach output voltage}}{R}$$

The advantage of this rating is that a complete calibration graph is not necessary.

5. *Find and record the output rating* of your tach generator, as indicated on the data page.

6. *Check two values of rotational speed* given by the relation above with the values from your graph.

EXPERIMENT A-4 (OPTIONAL). Using a Vibrating Reed Tachometer

In this experiment you can see how the vibrating reed tachometer works. Bolted to the side of the fan frame is a thin strip of spring steel, called a *reed*. It is similar to the reeds used in vibrating reed tachometers.

If pulled to one side and released, the reed vibrates back and forth at a certain rate or *frequency*. This "natural" frequency depends on the thickness and length of the reed as well as the material it is made of. The thinner the reed, the slower the vibrations; the longer the reed, the slower the vibrations. Thus, a long, thin reed has a much lower vibration frequency than a shorter, thicker reed.

When the reed is attached to the fan, the small vibrations of the fan cause the reed to vibrate. If you watch closely, you will see that the reed vibrates a little at any fan speed. However, when the fan's rotational speed (in rpm) is exactly equal to the reed's natural frequency of vibration (in vibrations per minute) the reed vibrates very wildly.

Follow the procedure below to see how this phenomenon is used to make a vibrating reed tachometer.

Procedure

1. *Adjust the free length* of the reed to about six inches. Clamp it firmly at this length. Snap the reed so that it vibrates back and forth.

2. *Measure the natural frequency* of the reed vibrations using the stroboscope. The same general rules for using the strobe to measure the rotational speed of a rotating object apply to measuring the vibration frequency of a vibrating object.

3. *Turn on the fan* and adjust its speed until the reed vibrations become very large. Since the fan speed is difficult to control with the voltage supply, it may help to use the strobe as a guide. Leave the strobe adjusted to the natural frequency of the reed and let it shine on the fan blade. Then adjust the fan speed until the apparent motion stops. This rotational speed should produce the vibration resonance in the reed. Does it?

   How nearly matched to the vibration frequency of the reed must the rotational frequency be to produce resonance? This is the basis for the use of a vibrating reed to measure a *single* frequency.

4. *Increase the length* of the reed by about an inch and repeat your measurements.

5. *Decrease the length* of the reed by two inches and repeat. How does the natural frequency of the reed seem to depend on its length?

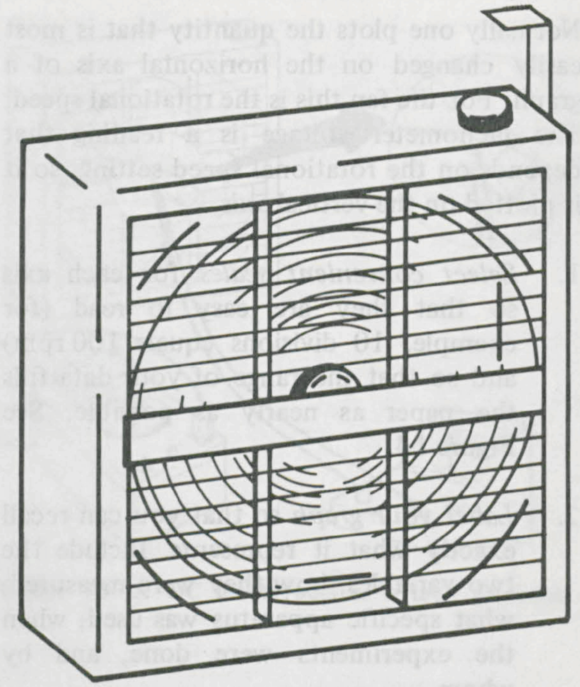


Figure 15. A simple strip of metal can act as a vibrating reed tachometer.

EXPERIMENT A-5. Measuring Rotational Acceleration

The turns counter, the stroboscope, and the vibrating reed can be used to measure constant or slowly changing rotational speeds. But none of these tachometers can follow rapid changes of rotational speed. The tach generator, on the other hand, gives an almost instantaneous reading of the rotational speed as it changes, just as an autombile speedometer reads the instantaneous speed of the car as it accelerates.

In this experiment you will explore the start-up behavior of the fan using the tach generator. As the fan *accelerates* from 0 rpm to its final rotational speed, you will measure the speed at different times. The *rotational acceleration* will vary as the voltage supplied to the fan is varied.

Procedure

1. *Set up the apparatus* used in Experiment A-3 as shown in the figure. Set the variable voltage supply at 110 V. You will need a stopwatch to record the time.

2. *Replace the fan blade with the metal disk* as shown. Bolt six heavy washers–three on each side of the disk–to the disk at the outer edge of each of the four slots. This added weight will cause the disk to be accelerated much more slowly than the fan blade, so you can make careful measurements. In Section B you will find out why this works.

3. *Make a table* in the space provided on the data page. You will need a column for rotational speed and one for elapsed time for each of several applied voltages.

4. *Measure the increasing rotational speed* of the fan blade at 110 V. Simultaneously switch on the fan and start the watch. Take readings of the tach voltage as often as you can, say every 5 s. Since the data must be taken quickly, it is best if two persons work together. You should make several trials to be sure your data are accurate.

5. *Repeat the measurement,* decreasing the voltage supplied to the fan 10 V at a time down to about 80 V.

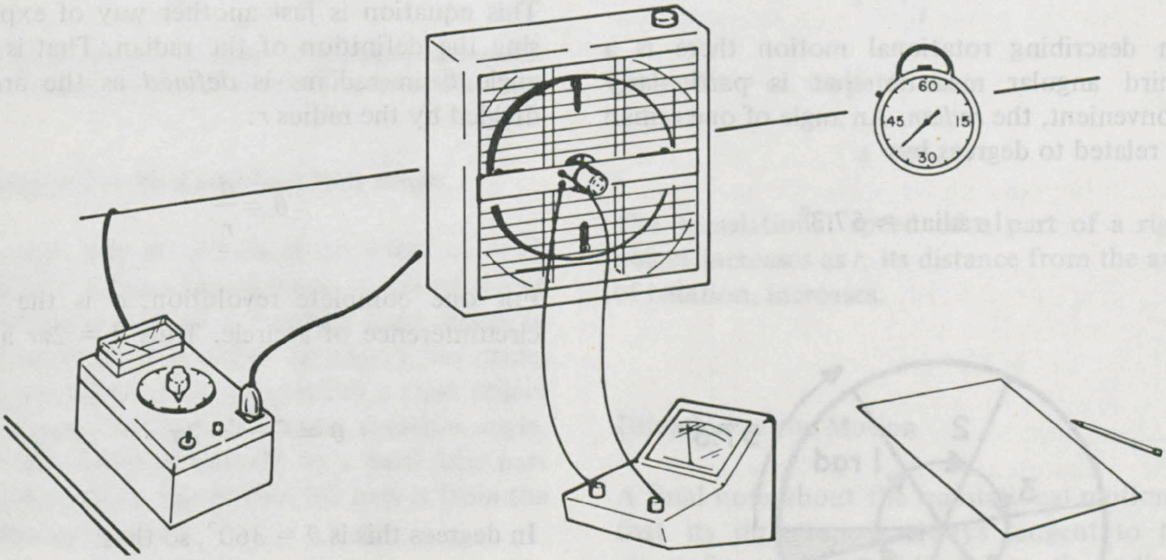


Figure 16. To study the start-up behavior of the fan, replace the blade with a metal disk with heavy washers attached.

THE DESCRIPTION OF ROTATION

Choosing a Scale for Angles

We have been using the number of revolutions to describe the total angle through which a rotating body has turned. One revolution means that the body has turned completely around once. Another way to measure rotation is to specify the angle through which the object turns in degrees. If you mark a reference line along a radius of a disc, one revolution means that the line has turned through an angle of 360°. One-fourth of a revolution is 90°, etc.

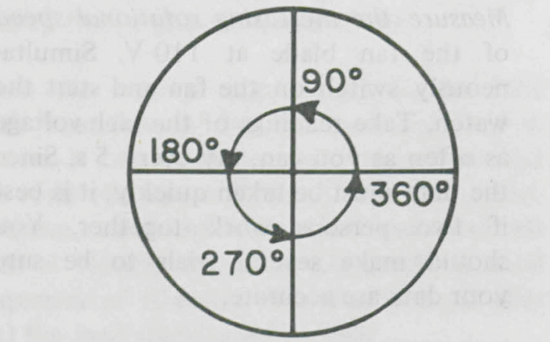


Figure 17. The geometrical measure of rotation angle is degrees.

The Radian

In describing rotational motion there is a third angular measure that is particularly convenient, the *radian.* An angle of one radian is related to degrees by:

$$1 \text{ radian} \approx 57.3°$$

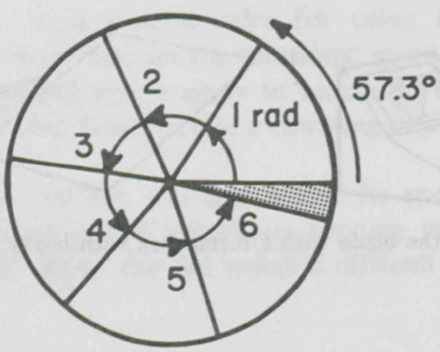


Figure 18. In rotational motion the radian is often a convenient measure.

Figure 18 shows how large a radian is. There are slightly more than six radians in a full revolution. In fact there are exactly 2π *radians* (about 6.28) *in one revolution.*

Relating Angles to Arcs

This number of radians per revolution is not accidental. It was introduced to simplify the relation between an angle and the corresponding distance along the circumference of a circle. If an angle θ (Greek letter *theta*) is measured in *radians,* then the distance d along the circumference is related to the radius r by:

$$d = r\theta$$

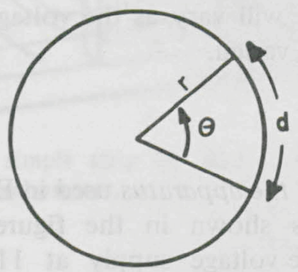


Figure 19. The arc is simply related to the radius if θ is measured in radians.

This equation is just another way of expressing the definition of the radian. That is, an angle θ in radians is *defined* as the arc d divided by the radius r:

$$\theta = \frac{d}{r}$$

For one complete revolution, d is the full circumference of a circle. Thus $d = 2\pi r$ and:

$$\theta = \frac{2\pi r}{r} = 2\pi$$

In degrees this is $\theta = 360°$, so that:

$$2\pi \text{ radians} = 360°$$

$$6.28 \text{ rad} = 360°$$

$$1 \text{ rad} \approx 57.3°$$

Motion of a Part of a Rotating Object

We can make a further distinction between various kinds of motion. For example, the earth is a rigid body which "rotates" (spins) on its axis. On the other hand, it "revolves" around the sun. The revolving motion about the sun is a *translational* motion along a nearly circular path.

Similarly, the motion of any particular *part* of a rotating rigid body is a translation along a circular path. Thus, it can be described in translational terms. Since each part is firmly attached to the rigid object, its translational motion is determined by the rotational motion of the object. Therefore, as indicated in Figure 20, the two motions are related.

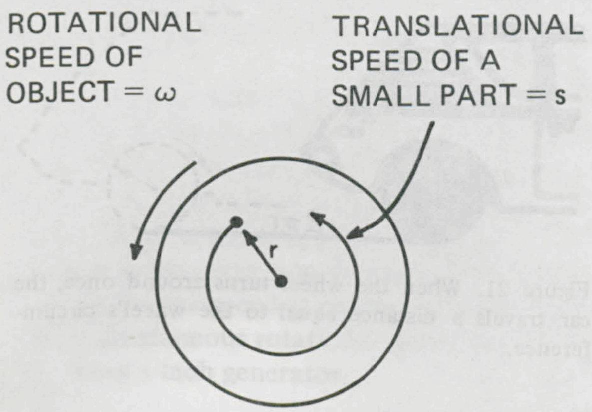


Figure 20. Motion of a part of a rotating rigid object is a translational motion around a circular path.

Distance Traveled and Rotation Angle

Another way of looking at the relation $d = r\theta$ is that it relates the distance, d, traveled in a circle by a part of a rotating rigid object to the amount of rotation the object has made, θ. (See Figure 19.) All parts of a rigid object will move through the same rotation angle, but the distance traveled by a particular part will be greater, the farther the part is from the center of rotation.

Translational Speed and Rotational Speed

The translational speed s of an object is the distance traveled d divided by the time t.

That is:

$$s = \frac{d}{t}$$

The rotational speed of a rigid object is defined by a similar expression relating the amount of rotation θ to the time for the rotation. Designating the rotational speed by the Greek letter *omega* (ω) we have

$$\omega = \frac{\theta}{t}$$

In Experiment A-2, for example, you measured θ in terms of the number of rotations, which gave ω in terms of revolutions per minute (rpm). If θ is measured in radians, then the rotational speed is in radians per minute.

In Figure 20, if s is the translational speed of some part of the object around a circular path and ω is the rotational speed of the rigid object (in radians per minute), then the two are related by the distance-angle expression $d = r\theta$. Substituting this into the expression for s gives:

$$s = r\frac{\theta}{t}$$

Since $\theta / t = \omega$, we get:

$$s = r\omega$$

The translational speed of a part of a rigid object increases as r, its distance from the axis of rotation, increases.

Direction of the Motion

A final note about the translational motion is that its direction is always tangent to the circumference (perpendicular to the radius). If a portion of a rotating rigid object should suddenly break off, it will go off at speed s in a direction tangent to the circumference at the point of release. This can be a danger for high-speed rotating devices.

HOW TO USE THE EQUATIONS

The Fan Blade

As an example of how to use these equations in calculations, let us apply them to the motion of the fan. Assume that the maximum rotational speed of the blade was measured to be 1500 rpm. Then answer the following questions:

1. *What is the maximum rotational speed in radians per second?*

We know that, in rpm:

$$\omega = 1500 \frac{\text{rev}}{\text{min}}$$

Since there are 60 s/min and 2π rad/rev,

$$\omega = 1500 \frac{\cancel{\text{rev}}}{\cancel{\text{min}}} \times \frac{1 \cancel{\text{min}}}{60 \text{ s}} \times 2\pi \frac{\text{rad}}{\cancel{\text{rev}}}$$

$$= 157 \text{ rad/s}$$

2. *What is the translational speed of a point on the fan blade a distance 6 in from the axis?*

We know the linear speed s in terms of the rotational speed ω.

$$s = r\omega$$

Substituting the above value for ω in rad/s and r in feet we get:

$$s = 0.5 \text{ ft} \times 157 \text{ rad/s}$$

$$= 78.5 \text{ ft/s}$$

The unit "rad" is dropped since it, like π, is a "pure" number with no units.

NOTE: Tables for converting between various units for angles and rotational speeds are provided on page 22. You should be able to figure out how each entry was determined.

An Automobile Odometer

The odometer of an automobile is essentially a turns counter. It is usually attached to a wheel of the drive shaft of the car through a flexible cable. When the wheel travels one-tenth mile of linear distance, the number of revolutions it has made causes the first digit of the odometer to increase by one.

3. *How many revolutions does a 20-in diameter tire make in traveling one mile?*

As the wheel turns, the distance traveled by a point on the tread is equal to the distance traveled by the car itself. (See Figure 21.)

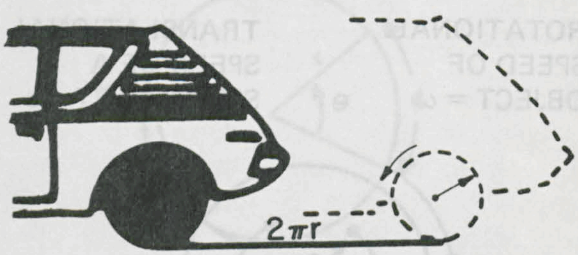


Figure 21. When the wheel turns around once, the car travels a distance equal to the wheel's circumference.

Using $d = r\theta$ with $d = 1$ mi $= 5280$ ft and $r = 10$ in $= 10/12$ ft:

$$d = r\theta$$

$$5280 \text{ ft} = \frac{10}{12} \theta \text{ ft}$$

$$\theta = 6336 \text{ rad}$$

$$= 1008 \text{ rev}$$

The odometer must have a set of gears with a gear ratio of about 1000 to 1 so that 1000 turns of the wheel produce 1 turn of the odometer.

CHANGING ROTATIONAL SPEED

Up to now our description of rotation has been limited to objects whose rotational speed is constant. But in Experiment A-5 the speed was steadily changing as it increased from zero to its final value. The term which describes changing rotational speed is *rotational acceleration.* It is generally designated by the Greek letter *alpha* (α).

Instantaneous vs. Average Rotational Speed

When an object has rotational acceleration, we must be more careful about how we specify its rotational speed. We distinguish between its *instantaneous* rotational speed and its *average* rotational speed.

The *instantaneous* rotational speed is the rotational speed at any specific "instant" of time. It is comparable to the instantaneous speed of a car, which is read by the speedometer. Even when the car is accelerating, the speedometer gives you a reasonably accurate reading of the speed at each instant. Similarly, the instantaneous rotational speed can be read by using a tach generator.

The *average* rotational speed of an object is what you measured in Experiment A-1. You had no idea what the rotational speed was at any moment or whether it was changing. But you did know how far it had rotated (the total number of turns) in a given length of time, and you could calculate the average rotational speed.

This is similar to determining your average automobile speed by dividing the total number of miles you went by the number of hours it took. This works even though you may have had a lot of starts and stops.

If the angular speed is *constant,* then the average and instantaneous speeds are the same.

Graphing Changes in Rotational Speed

Rotational acceleration is difficult to measure directly. Instead one usually draws a graph of the speed as it changes with time, then *calculates* the acceleration from the graph.

Graph the data of Experiment A-5. First convert the tach-voltage readings to rotational speed in rpm, using your calibration graph or the output rating. Plot the data for different voltages on the same piece of graph paper. To identify the points corresponding to different voltages, draw different shapes around them, such as ⊙, ⊡, ⧍, X.

Draw a smooth curve through the points. Do not just connect the points with straight-line segments; the fan speed changes smoothly, not erratically. If possible, use a draftsman's French curve to help draw a smooth graph of the changing rotational speed.

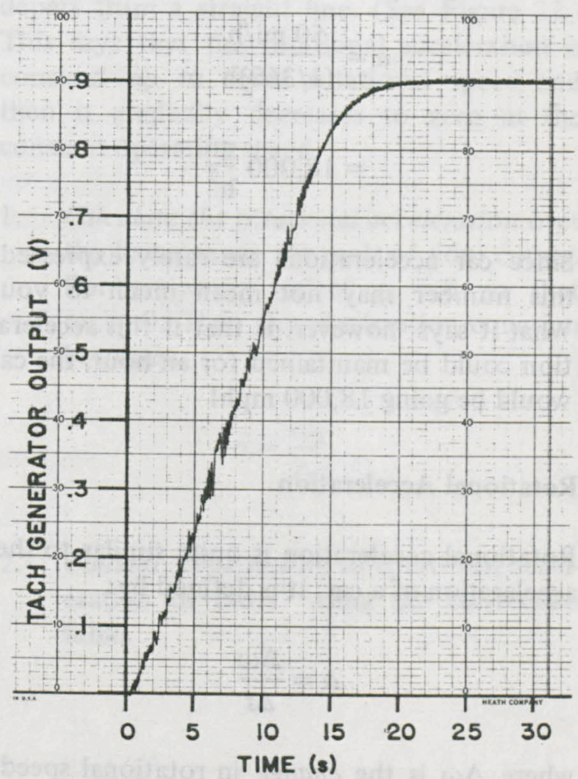


Figure 22. Typical start-up behavior of a fan. This curve was made by using a chart recorder.

ACCELERATION

The rotational acceleration α is a measure of the *rate* at which the rotational speed is changing. Just as with an automobile's motion, if the acceleration is large, then the speed is changing rapidly; if it is small, the speed is changing slowly. If there is no acceleration, the speed is constant.

Linear Acceleration

For an automobile, the average acceleration a is determined by dividing the change in speed Δs* by the time Δt over which the change occurs:

$$a = \frac{\Delta s}{\Delta t}$$

For example, if a car accelerates from 0 to a speed of 50 mph in 10 s ($\Delta t = 1/360$ h), then the acceleration is:

$$a = \frac{50 \text{ mi/h}}{(1/360)\text{h}}$$

$$= 18{,}000 \ \frac{\text{mi}}{\text{h}^2}$$

Since car accelerations are rarely expressed, this number may not mean much to you. What it says, however, is that if this acceleration could be maintained for an hour, the car would be going 18,000 mph!

Rotational Acceleration

Rotational acceleration is quite similar to the acceleration of a car. It is defined by:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

where $\Delta\omega$ is the *change* in rotational speed.

*The Greek letter *delta* (Δ) is used in front of a quantity to indicate a change in that quantity.

We emphasize "change" because if the speed is not zero to start with, then the expression should be:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

where ω_1 is the rotational speed at the time t_1 and ω_2 is the speed at t_2.

Thus $\omega_2 - \omega_1$ is the "change" in speed during the time period $t_2 - t_1$.

Calculating Rotational Acceleration

The rotational acceleration can be found from a graph of ω against t. For any two times t_1 and t_2 on the curve, find the corresponding values of ω_1 and ω_2. The *average* rotational acceleration during that time interval is:

$$\frac{\omega_2 - \omega_1}{t_2 - t_1}$$

This equation defines the *slope* of the straight line connecting points 1 and 2 on the graph of Figure 23A. The slope is the vertical distance (the *rise*) $\omega_2 - \omega_1$ divided by the horizontal distance (the *run*) $t_2 - t_1$.

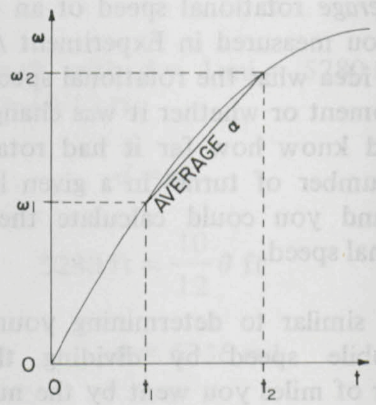


Figure 23A. The average rotational acceleration is the slope of the line connecting the two points.

To find the *instantaneous* rotational acceleration at time t_2, the same procedure is used. However, now the point t_1 is moved closer and closer to t_2, as indicated in Figure 23B.

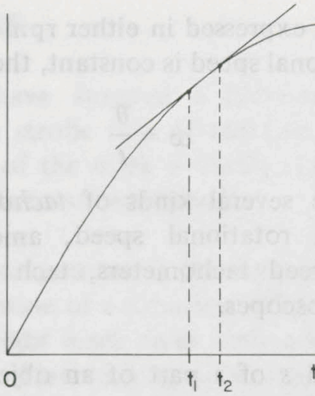


Figure 23B. As the time interval becomes smaller, the line connecting the points becomes the slope of the curve.

When t_1 gets so close to t_2 that they are essentially the same time, the straight line that touches the curve only at that single point is the *tangent* to the curve, as indicated in Figure 23C.

The slope of a curved line at a point on the line is defined to be the same as the slope of the straight line tangent to the curve at that point.

The slope of the curve at any time is the *instantaneous* rotational acceleration at that time. A positive slope means a positive acceleration and an increasing rotational speed. A negative slope means a negative acceleration (deceleration) and a decreasing rotational speed. A horizontal slope means no acceleration and constant rotational speed.

If the slope of a particular ω vs. t curve is constant during a time interval, then the angular acceleration is constant during that time. This means that the rotational speed is changing uniformly. If the slope is changing, however, then both the rotational speed *and* the rotational acceleration are changing.

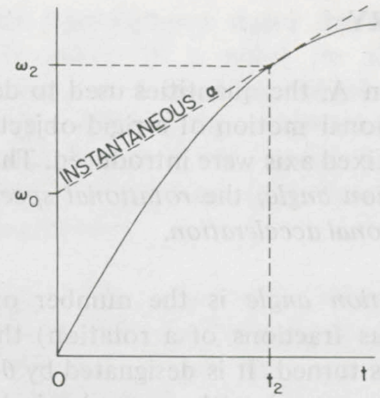


Figure 23C. The instantaneous acceleration is the slope of the curve at that instant of time.

Calculations from Your Graph

Your graph of the start-up behavior should be a straight line over most of the acceleration period. That is, it should have a constant slope. Only when the disk is almost up to its final operating speed does the curve start to depart from a straight line. (See Figure 22.) This says that the rotational acceleration is constant up to some rotational speed and then it gradually decreases to zero at the constant operating speed.

1. *Calculate the rotational acceleration* over the linear range for each of your curves. For simplicity select $t_1 = 0$ so that $\omega_1 = 0$. Then the acceleration equation becomes:

$$\alpha = \frac{\omega_2}{t_2}$$

2. *Convert your results* to rotational acceleration in rad/s^2 using the conversion table.

SUMMARY

In Section A, the quantities used to describe the rotational motion of a rigid object rotating on a fixed axis were introduced. These are the *rotation angle*, the *rotational speed*, and the *rotational acceleration.*

The *rotation angle* is the number of rotations (plus fractions of a rotation) that the object has turned. It is designated by θ and it can be measured with a mechanical turns counter.

Angles can also be measured in *radians.* If d is the distance along the circumference of a circle of radius r, then the corresponding θ in radians is:

$$\theta = d/r$$

The distance d traveled by a point on an object that has rotated through an angle θ can be calculated from:

$$d = r\theta$$

where r is the radius from the axis of rotation to the point.

Rotational speed is the rate at which an object is rotating. It is commonly designated by ω and expressed in either rpm or rad/s. If the rotational speed is constant, then:

$$\omega = \frac{\theta}{t}$$

There are several kinds of *tachometers* for measuring rotational speed, among them vibrating reed tachometers, tach generators, and stroboscopes.

The speed s of a part of an object rotating with rotational speed ω can be calculated from:

$$s = r\omega$$

where r is the radius from the axis of rotation to the part. The direction of the motion is always tangent to the circumference.

Rotational acceleration is the rate of change of rotational speed. It is designated by α and expressed in rad/s^2. It can be calculated from a graph of rotational speed versus time. At any time, the instantaneous rotational acceleration is the slope of the curve at that time. Mathematically, the average rotational acceleration is:

$$\alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

CONVERSION TABLES

ROTATION ANGLE:

To Go From...	To...	Multiply By
revolutions	radians	6.28
revolutions	degrees	360
radians	revolutions	.159
radians	degrees	57.3
degrees	revolutions	.00278
degrees	radians	.0174

ROTATIONAL SPEED:

To Go From...	To...	Multiply By
rev/s	rev/min	60
rev/min	rev/s	.0167
rev/min	rad/s	.105
rad/s	rev/min	9.55
rev/s	rad/s	6.28
rad/s	rev/s	.159

QUESTIONS

1. You have stopped a rotating motion with a strobe rate of 100 fpm. A single image of the mark is visible. List several possible rotational speeds.

2. Suppose you use a stroboscope to stop the motion of a rotational wheel that has one bright mark on it. Only one image is visible. How can you be sure that the strobe rate equals the rotational speed?

3. What is the angle turned (in radians) when a point on a wheel of radius 1.7 ft moves a distance of 3.4 ft? What is the angle in degrees?

4. A technician turns a control knob through an angle of 1°. How far does a pointer on the rim (radius 3/4 in) move?

5. Snow tires have a somewhat larger diameter than regular tires. Does the odometer read more than or less than one mile when the car with snow tires on travels one mile?

PROBLEMS

1. An engine flywheel is turning at 4000 rpm. The radius of the wheel is 9 in. What is the speed of a point on the rim in ft/s? ft/min? mi/h?

2. One revolution of a light airplane propeller takes 0.03 s. The radius of the blade is 3 ft. Find the rotational speed and the linear speed of a point on the tip of the blade.

3. What is the rotational speed (rad/s) of a tire of 18-in radius when the car is moving 150 mi/h (88 ft/s = 60 mi/h)?

4. The manufacturer states the maximum safe speed of a point on the outer surface of a grinding wheel is 4000 ft/min. What is the maximum safe rotational speed if the wheel is of 6-in diameter? Calculate this, first in rad/s; then in rpm.

5. What is the speed of a 78-rpm record beneath the tone arm of a record player if the head is 6 in from the axis? 3 in from the axis?

6. Figure 24 shows typical slow-down behavior of a fan. Calculate the instantaneous rotational acceleration at $t = 5$ s. Calculate the average acceleration between 5 s and 25 s. (Since the fan is slowing down, these rotational accelerations will be negative.)

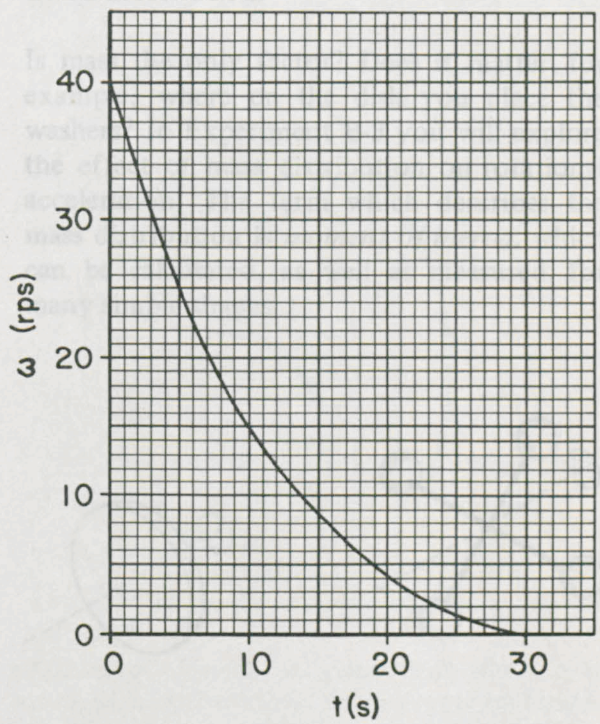


Figure 24.

SECTION B

Explaining Rotational Motion

WHAT DETERMINES ROTATIONAL ACCELERATION?

In Experiment A-5 you found that by decreasing the voltage supplied to the fan, you lengthened the time required to accelerate the disk to its constant final speed. A decrease in the voltage decreased the ability of the motor to cause rotational acceleration. Somehow, the "strength" of the motor is related to the voltage.

The purpose of Section B is to understand rotational "strength" and how it causes rotational acceleration. To do this, we need both a term to describe rotational strength and a way of measuring it. The term used is *torque.*

Torque

The torque of a motor is simply a measure of how strong a "twist" it can apply to an object to produce rotational acceleration. In Experiment B-1, for example, you will measure the *stall torque* of the fan motor. This is the amount of twist you have to apply to keep the motor from rotating, that is, to overcome the rotational torque of the motor.

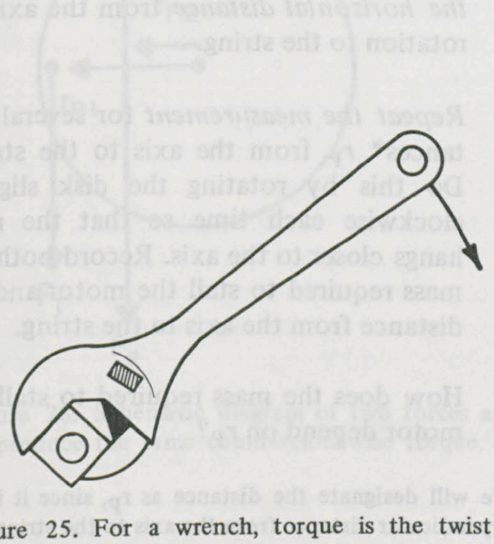


Figure 25. For a wrench, torque is the twist produced by a force applied to the end of the wrench.

In Experiment B-2 you will supply a torque which you can measure and see how much rotational acceleration it produces. You then will be able to determine the *motor torque* for various supply voltages by using the data you took in Experiment A-5.

Moment of Inertia

In Experiment A-5 you replaced the fan blade with a disk that could hold extra masses. These extra masses slowed the rate of rotational acceleration so that you could measure it. This slowing shows that the mass of the object being rotated affects the rate of rotational acceleration.

Is mass the only factor? Does it matter, for example, where on the disk you place the washers? In Experiment B-3 you will explore the effect of mass distribution on rotational acceleration. The term which describes the mass distribution is *moment of inertia,* which can be calculated, as well as measured, for many simple shapes.

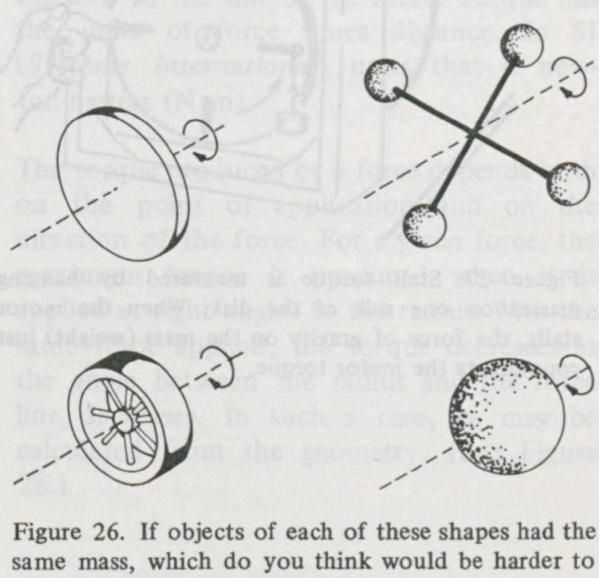


Figure 26. If objects of each of these shapes had the same mass, which do you think would be harder to turn?

EXPERIMENT B-1. Measuring Torque

In this experiment you will measure the *stall torque* of the fan motor. This is an important specification for motors since it tells how big a load the motor can overcome without stalling. If a motor stalls, it may overheat and burn out unless properly protected.

To measure stall torque, load one side of the disk with masses. The weight of the masses produces a counter-torque to the torque of the motor. An important part of the measurement is to determine how the force required to stall the motor depends on its distance from the axis of rotation. This dependence is the key to an understanding of torque.

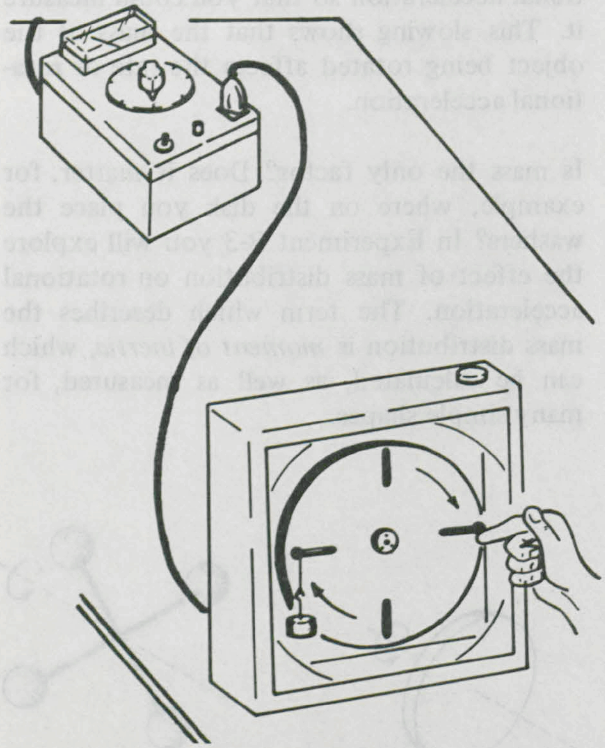


Figure 27. Stall torque is measured by hanging masses on one side of the disk. When the motor stalls, the force of gravity on the mass (weight) just counteracts the motor torque.

Experimental Procedure

1. *Set up the fan* as shown in Figure 27. Remove the protective grill and make sure that the disk is mounted securely on the shaft. Turn the variable voltage supply to a maximum, but be sure it is off. Mount identical bolts through the slots on opposite sides of the disk, as shown.

2. *Attach a mass* to one of the bolts with a string so that it hangs as shown.

3. *Turn on the voltage* while holding the disk to keep it from turning. It is important to orient the disk so that the slots are horizontal.

4. *Add a hanging mass* until the fan motor is *just* able to overcome the torque produced; that is, until it can barely turn the disk. You will have to switch the voltage on and off quickly, because the disk will turn if the mass is not large enough.

   CAUTION: *Do not let the motor stall for more than a few seconds at a time.*

5. *Record the mass* required to stall the motor in the data table. Also *measure the horizontal distance* from the axis of rotation to the string.

6. *Repeat the measurement* for several distances* r_P from the axis to the string. Do this by rotating the disk slightly clockwise each time so that the mass hangs closer to the axis. Record both the mass required to stall the motor and the distance from the axis to the string.

   How does the mass required to stall the motor depend on r_P?

*We will designate the distance as r_P, since it is the perpendicular distance from the axis to the string.

Simplifying the Behavior

To better understand what was happening in Experiment B-1, we must simplify the situation as much as possible. Figure 28 accomplishes this simplification by showing only the forces and points of application for two different orientations of the disk.

The torque of the motor trying to rotate the disk is indicated by the Greek letter *tau* (τ). Since torque is a twisting effect, it is shown as a curved arrow in the direction of the twist.

Gravity exerts a force on the hanging mass which is straight down, so its effect on the disk is shown as an arrow pointing down. The arrow starts at the point of application, the bolt, and its length is proportional to the force exerted by the weight hanging on the string.

1. *Calculate the gravitational force* exerted on each of the masses used in Experiment B-1. The gravitational force in newtons (N) is the mass (in kg) times the gravitational acceleration g (9.8 m/s^2):

$$F = mg$$

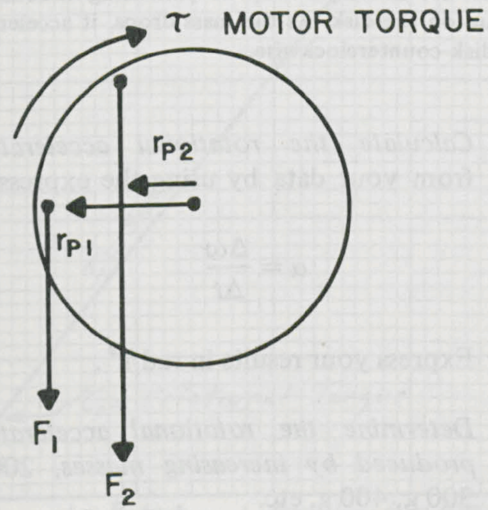


Figure 28. Schematic diagram of two forces acting to produce the same counterclockwise torque.

Calculating the Torque

Now the question becomes, what is the relation between the force exerted on the disk by the hanging mass and the torque exerted by the motor? We know that when the motor stalls, they produce equal but opposite results which cancel.

Also, as the line along which the force is applied moves closer to the axis of rotation, more force (greater weight) is required to stall the motor. To account for this fact, multiply the force required to stall the motor by the horizontal distance of the force arrow from the axis.

2. *Multiply each force (weight) by its distance* r_P from the axis of rotation. Be sure to express r_P in meters.

Now look at your results. You should see that each product is approximately the same. This suggests that the product Fr_P is the amount of counter-torque required to stall the motor.

As you may have gathered, this is actually the relation between any applied force F and the amount of torque τ it produces. That is:

$$\tau = Fr_P$$

where r_P is the perpendicular distance from the axis to the line of the force. Torque has the units of force times distance. In SI (*Système International*) units that is newton-meters (N·m).

The torque produced by a force depends both on the point of application and on the direction of the force. For a given force, the maximum torque is produced when it is applied at right angles to the radius. With the same force applied, the torque decreases as the angle between the radius and the force line decreases. In such a case, r_P may be calculated from the geometry. (See Figure 28.)

EXPERIMENT B-2. Torque and Rotational Acceleration

By loading one side of the disk you can produce a torque that will equal or overcome the torque supplied by the motor. Also, a given force produces the most torque when it is applied perpendicular to the radius of the disk.

You can use these results to explore the relation between an applied torque and the amount of rotational acceleration it produces. Figure 29 shows a simple way of applying a constant torque to the fan. Wrap a string around the hub and hang a weight on it. As the weight falls, it pulls on the disk and causes it to accelerate. Notice that the string is also always at right angles to the radius, so the torque produced by the weight is constant during the whole process.

Experimental Procedure

1. *Set up the arrangement shown in Figure 29.* The fan should be clamped firmly to the table and turned off. *Remove all bolts and washers* from the disk.

   Tie a string to the hub of the disk so that it hangs freely through the bottom of the fan. There should be a way of attaching masses to the string and the masses should be able to drop for several feet.

   Attach the tach generator so you can measure the rotational speed of the disk.

2. *Measure and record the radius* of the hub that the string wraps around.

3. *Measure the rotational acceleration produced by a 100-g mass.* Simultaneously release the mass and start the stopwatch. When the mass hits the floor, stop the watch and note the reading on the tach generator. This will probably require two people and some practice. For better accuracy repeat the procedure at least three times and record the *average* of the three measurements.

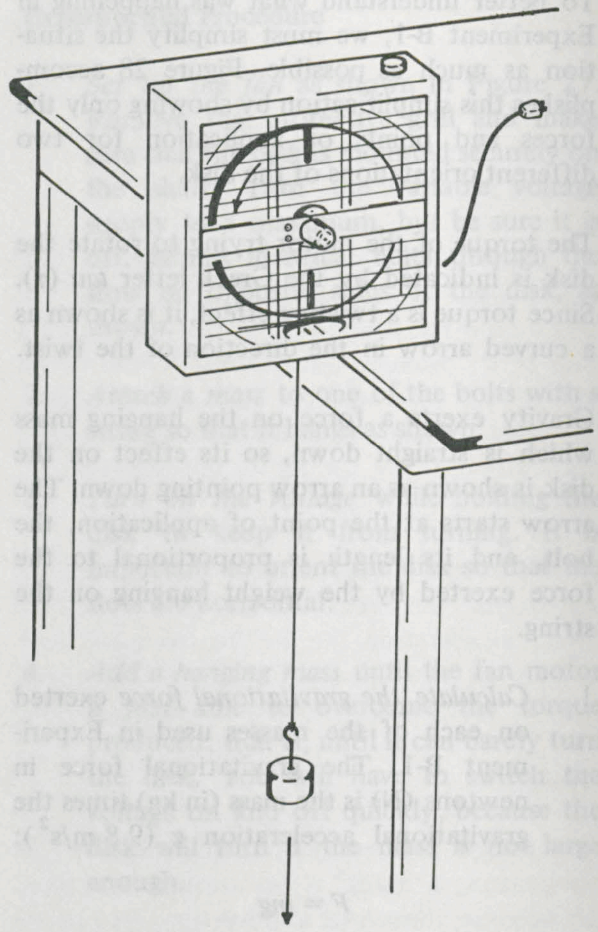


Figure 29. A simple way of producing a constant torque on the disk. As the mass drops, it accelerates the disk counterclockwise.

4. *Calculate the rotational acceleration* from your data by using the expression

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Express your results in rad/s^2.

5. *Determine the rotational acceleration produced by increasing masses,* 200 g, 300 g, 400 g, etc.

Plotting the Data

The goal of this experiment is to determine the relation between an applied torque and the rotational acceleration it produces. The

simplest way to find this is to make a graph of your data. Follow the procedure below to make your graph.

1. *Calculate the torques produced* by the various masses suspended from the hub. As before, the force is the mass (in kg) times the gravitational acceleration 9.8 m/s^2.*

2. *Graph your data.* Place torque on the vertical axis and rotational acceleration on the horizontal axis.

You should find that the points lie pretty nearly on a straight line. If the apparatus were more carefully designed and your measurements were extremely accurate, you would find that the points lay exactly on a straight line.

*With the hanging mass accelerating, this is not quite accurate. However, for our purposes the correction is very small and can be ignored.

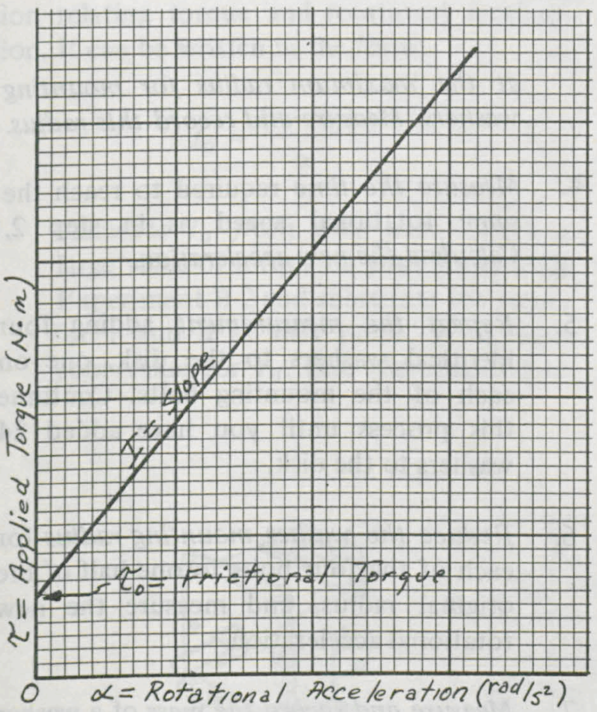


Figure 30, The graph of the data of Experiment B-2 should be similar to this.

3. *Draw the best straight line* you can through your data points. The "best" straight line means the one for which as many points lie above as below, and at about equal distances. You should find that the best line does not go through the origin.

Interpreting the Results

The fact that a graph is a straight line means that changes of the two graphed quantities are proportional. Increase one and you increase the other. If we call the proportionality constant *I*, then we can describe the proportionality mathematically as:

$$\tau = I\alpha$$

The constant *I* is the *slope* of a straight-line graph.

However, this equation does not quite describe your results. It says that when $\tau = 0$, $\alpha = 0$, which, for your data, is not the case.

Determining the Frictional Torque

Your graph implies that there is no rotational acceleration ($\alpha = 0$) until the applied torque reaches some minimum value, τ_0. Another way of saying this is that you have to apply a torque τ_0 just to overcome the friction of the motor. Thus we call τ_0 the *frictional torque.*

4. *Read and record the frictional torque* of your fan motor from the graph.

Now, if we subtract the frictional torque from the applied torque, the resulting torque $(\tau - \tau_0)$ is directly proportional to the acceleration produced:

$$(\tau - \tau_0) = I\alpha$$

Now the question is: what does *I* represent? In the next experiment you will explore the meaning of the constant *I.*

EXPERIMENT B-3. Rotational Characteristics of Rigid Objects

The final thing to explore is the way in which the rotational acceleration depends on what is being rotated. That is, for a given torque–for example, that of the fan motor–how does the acceleration depend on the mass of the disk? You will investigate this by measuring the disk's acceleration as you load it with more and more heavy washers.

You should also realize that the motor must accelerate its own rotor, in addition to the disk. You can neither see nor weigh the rotor. How do you think that its effect can be taken into account?

Finally, you will see what effect the *distribution* of washers has on the acceleration rate. Does it matter, for example, whether they are far out on the edge or in close to the axis? If it does matter, then by how much? If you halve the radius, for example, would you expect the acceleration rate to double? Triple? Quadruple?

Experimental Procedure

1. *Set up the fan* as shown in Figure 31. The disk is identical to that used in Experiment B-2 (that is, with no masses attached).

   Turn the supply voltage to 90 V.

   Attach the tach generator to measure the rotational speed.

2. *Measure the rotational acceleration of the disk.* Simultaneously start the fan and the stopwatch, and measure the time required for the disk to reach some rotational speed in the constant acceleration range, say 1300 rpm.

   Calculate and record the rotational acceleration.

3. *Add four identical bolts in the four slots at the maximum radius for mounting washers. Measure and record this radius.*

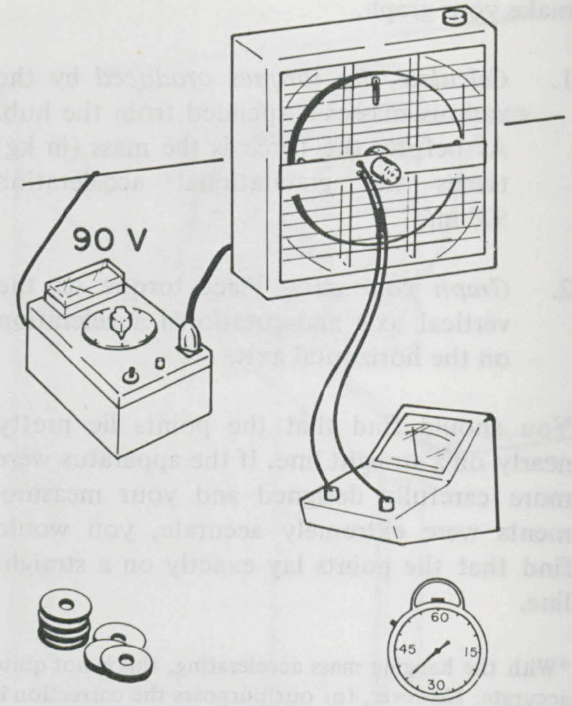


Figure 31. Here you will measure the acceleration produced by the motor as you change the mass of the disk.

4. *Measure the time* required to reach the *same* rotational speed as in step 2. *Calculate the new acceleration.*

5. *Repeat the measurement* adding four identical washers to the disk, one on each of the mounting bolts. Continue this process until you have added 24 washers to the disk.

6. *Reduce the washer mounting radius* for each of the four bolts to one-half of the original radius, and measure the new rotational acceleration.

7. *Measure and record the mass* of a washer and of a mounting bolt. Verify that all the washers and all the bolts are identical.

Interpreting the Results

Your results should show that as you increase the mass of the disk, the acceleration the motor is able to produce gets smaller and smaller. You also saw in step 6 that mass is not the only factor. When you decrease the mounting radius and keep the mass constant, the acceleration increases.

Since torque is proportional to rotational acceleration, you must have changed the constant of proportionality *I*. That is, *I* must in some way represent the rotational characteristics of the object. It must include both the mass of the object and where the mass is located.

Because the disk rotating in Experiment B-2 was the same as that for step 2 of Experiment B-3, the value of *I* for it (which we will call I_0) was the same for the two experiments. If we can calculate I_0 from Experiment B-2, then we can use it in Experiment B-3. I_0 can be calculated from the mathematical expression relating torque and rotational acceleration. It can be written in the form:

$$I_0 = \frac{(\tau - \tau_0)}{\alpha}$$

1. *Calculate I_0 from your data.* Select a large value of torque on your graph from Experiment B-2 and substitute the values for τ and α at that point, along with your value for τ_0, into the equation to get I_0. Your result should have units of $(\text{N}\cdot\text{m})/(\text{rad/s}^2)$. We will simplify this unit later.

Calculating the Motor Torque

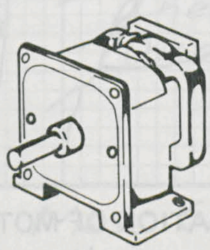
The value I_0 represents the rotational characteristics of the disk with no masses attached. It includes the rotor, axle, and anything else inside the motor that was rotating. This quantity is a constant, and it does not change for step 2 of Experiment B-3. Therefore, we can write for B-3:

$$\tau = I_0\alpha$$

where τ is the motor torque at 90 V and α is the acceleration you measured. Since you know I_0 and α you can now determine the motor torque.

2. *Calculate the motor torque* for 90 V. Be sure you have converted the rotational acceleration of step 2 to rad/s^2 by multiplying rpm's by 2π.

This value represents the torque supplied by the motor when it is rotating. It is called the *dynamic* or *full-load torque* to distinguish it from the *stall torque* you measured in Experiment B-1. The stall torque is also referred to as the *starting torque* since it is the torque required to overcome the initial load to get the rotation started (See Figure 32.) (Note that the motor torque is the actual torque capability of the motor less the frictional torque, τ_0.)



PERFORMANCE CHARACTERISTICS

Typical Standard 115-V 60-Hz AC Unidirectional Geared Motors

GEAR RATIO	NO LOAD			STARTING		FULL LOAD				
	Amps	Watts	rpm	Watts	Torque lb - in	Torque lb - in	rpm	hp	Amps	Watts
15:1	.13	7	200	8	.42	.30	120	.00061	.13	8
30:1	.13	7	100	8	.80	.60	60	.00056	.13	8
60:1	.13	7	50	8	1.5	1.2	30	.00056	.13	8
120:1	.13	7	25	8	2.8	2.0	15	.00048	.13	8

Figure 32. Performance characteristics of a typical motor used in vending, office, and copy machines. Note that the starting torque is greater than the full-load torque.

THE EQUATION OF MOTION

The equation that describes the behavior of the disk in Experiment B-2 is, in fact, the basic equation of rotational motion. It gives the amount of rotational acceleration α that an applied torque τ will produce on an object whose rotational characteristic is I.

$$\tau = I\alpha$$

It is the rotational equivalent of *Newton's second law* which determines translational motion:

$$F = ma$$

In Newton's second law F is the net force applied to an object whose translational characteristic is its mass m. This force will produce an amount of translational acceleration a.

Torques are caused by forces applied to an object so as to cause rotation about an axis. The ability of a force to produce a rotational acceleration is determined by the torque and by I. The torque depends upon both F and the distance r_P.

The rotational characteristic I is called the *moment of inertia.* It determines the amount of rotational acceleration that a torque will produce in much the same way that the mass of an object determines the amount of translational acceleration that a force will produce.

In Experiment B-3, you explored moment of inertia by applying a known torque to the disk and measuring the change in rotational acceleration as you changed the disk's properties. There were two factors that you changed, the total *mass* and the *radius* at which some of the mass was located.

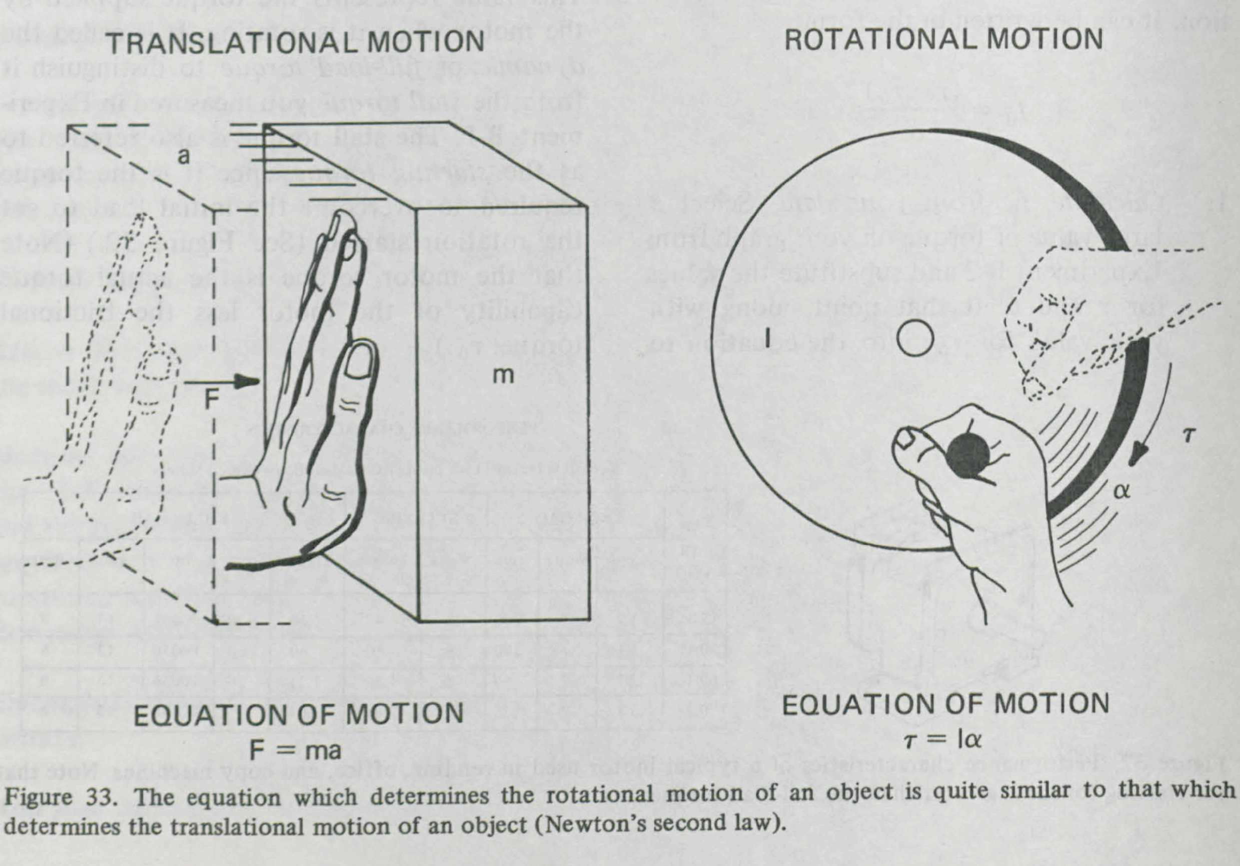


Figure 33. The equation which determines the rotational motion of an object is quite similar to that which determines the translational motion of an object (Newton's second law).

MOMENT OF INERTIA

To begin our discussion of moment of inertia, let us explore the first factor, mass. As usual, the first step is to graph the data. In this case we want to see how the moment of inertia I changes with mass. To do this, you should plot I against the amount of mass that you added to the disk m.

1. *Prepare a graph of I against m.* Since you have already calculated the motor torque at 90 V, you can use this value and the measured rotational acceleration to get I ($= \tau/\alpha$). Be sure to use τ in N•m and α in rad/s^2.

   The mass m should be in kg. This is the total mass that was added to the bare disk. It includes the mass of the bolts and nuts plus the mass of the washers added in each case.

You should find that once again the data points lie nearly on a straight line.

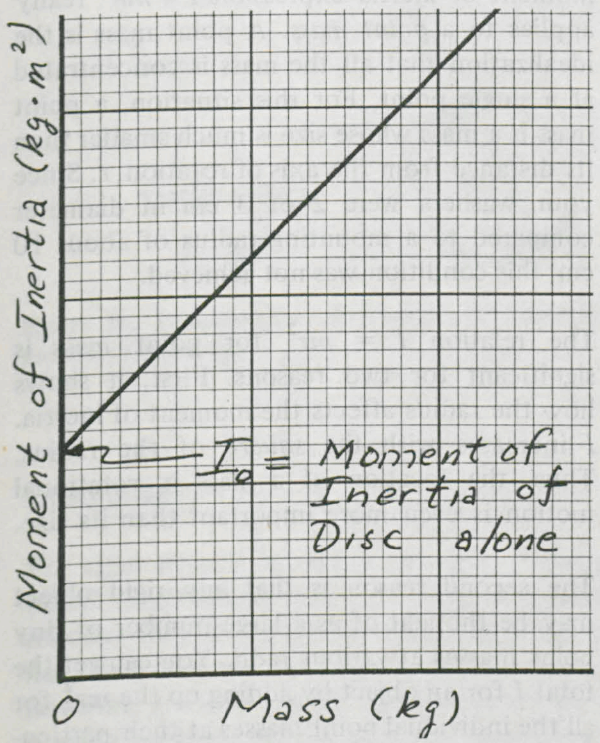


Figure 34. Your graph of moment of inertia of the disk vs added mass should be a straight line that does not go through 0.

2. *Draw a straight line through your points.* Recall that the best line has as many points lying above the line as below. Again your line will not go through the origin.

It is significant that the line does not go through the origin. It means that, without any additional mass, the disk has a moment of inertia equal to the value where the line intersects the I axis. This is what you previously calculated as I_0. The calculated value of I_0 should be approximately the same as the value from your graph. Is it?

Dependence on Mass

The fact that your points lie on a straight line means that changes in I are proportional to changes in m. The constant of proportionality is the *slope* of the line. In order to gain a better understanding of what this particular slope means, let us look at the units of I. From page 31, I ($= \tau/\alpha$) has units of:

$$I = \frac{\text{N}\cdot\text{m}}{\text{rad/s}^2}$$

Newtons (N) are force units. Since force is mass times acceleration, newtons are equivalent to kg $\times$ (m/s^2). Thus the units for I are:

$$I = \frac{\text{kg(m/s}^2) \times \text{m}}{(\text{rad/s}^2)}$$

The 1/s^2 in the numerator and denominator cancel, and the dimensionless rad can be dropped to give:

$$I = \text{kg}\cdot\text{m}^2$$

Thus I has units of mass times length squared.

Dependence on Radius

Since the graph indicates that I is proportional to mass (kg), the constant of proportionality (the slope) must have units of length squared (m^2).

3. *Calculate the slope of your graph.*

$$\text{Slope} = \frac{I_1 - I_0}{m_1 - 0}$$

where I_1 is a large value of torque, m_1 is the corresponding value of mass, and I_0 is the value of I at $m = 0$.

4. *Take the square root of your result.* This will give you a length in meters.

5. *Compare this length with the washer-mounting radius.*

You should find that the two values are nearly identical. What does this mean? If you assume that the slope of the line *is* the square of the radius for the added mass, then the following equation describes your straight line.

$$(I - I_0) = mr^2$$

This says that if you add a mass m to a rotating rigid object at a distance r from its rotation axis, you will increase its moment of inertia $(I - I_0)$ by an amount mr^2.

You can check this result by using the acceleration data you took when you changed r by moving the washers toward the center of the disk.

6. *Calculate I for the mounting radius used in step 6* of Experiment B-3, using the above equation.

7. *Compare your result* with the I you measured. You should find them to be nearly the same.

This expression also says that the total moment of inertia of the disk plus the washers is the sum of that of the disk alone plus that of the added washers alone. This is a quite general result. The moment of inertia of a combination of two objects is simply the sum of their individual moments of inertia about the same rotation axis.

$$I = I_0 + mr^2$$

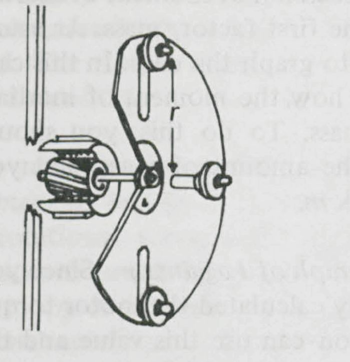


Figure 35. The moment of inertia of the rotating object is the sum of the moments of inertia of its parts.

Significance of the Point Mass

The values of I computed in steps 5 and 7 probably did not agree exactly with the measured values. One reason is that the moment of inertia expression $I = mr^2$ really applies to a *point mass*. A point mass is the idealization that all the mass is concentrated at a single point. For this situation, a point mass is a mass whose size is much smaller than its distance from the axis of rotation, r. Since your washers were 2 or 3 cm in diameter compared to a mounting radius of about 10 cm, this condition was not achieved.

The relation $I = mr^2$ for point mass is significant for two reasons. First, it shows how the radius affects the moment of inertia. I increases with the *square* of the radius. Thus, the location of a mass in rotational motion is even more important than its size.

The second reason is that any rigid object may be thought of as a large number of tiny point masses at various radii. You can get the total I for an object by adding up the mr^2 for all the individual point masses at their particular radii.

Dumbbells and Rings

This adding-up process is complicated for most shapes, but it is easy for some simple ones. For a dumbbell, a double dumbbell, or a thin ring, for example, we can assume that all the mass is at nearly the same radius. If each point mass is m, and the total mass is M, then the sum of all the mr^2 is Mr^2. (See Figure 36.)

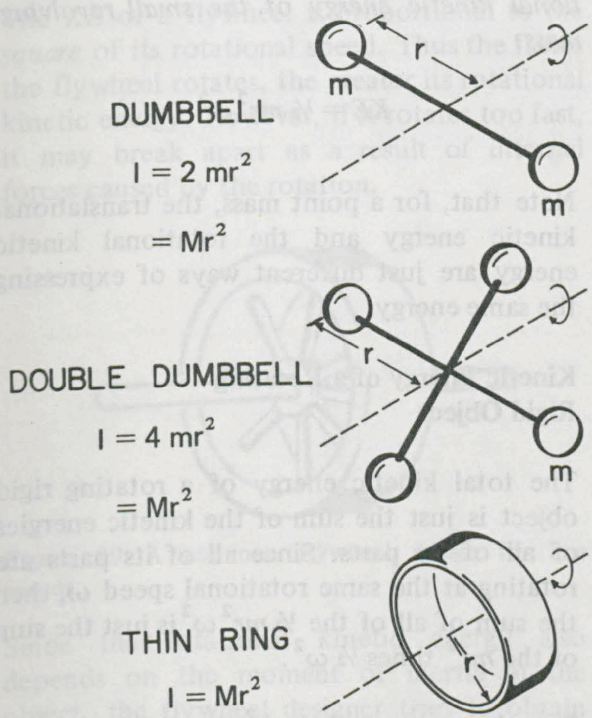


Figure 36. Equations for the moment of inertia of dumbbells and rings.

Other Shapes

For rigid objects in which the mass is distributed at different radii, the adding process requires the mathematical technique of calculus. The results for some simple shapes, however, are quite simple. Figure 37 shows a number of these shapes with the corresponding expressions for I.

For each, the moment of inertia includes the *total mass* times a *length squared*. If the mass is not all at the same radius, there is a reducing fraction out front that takes this into account. For the disk, for example, the fraction is ½.

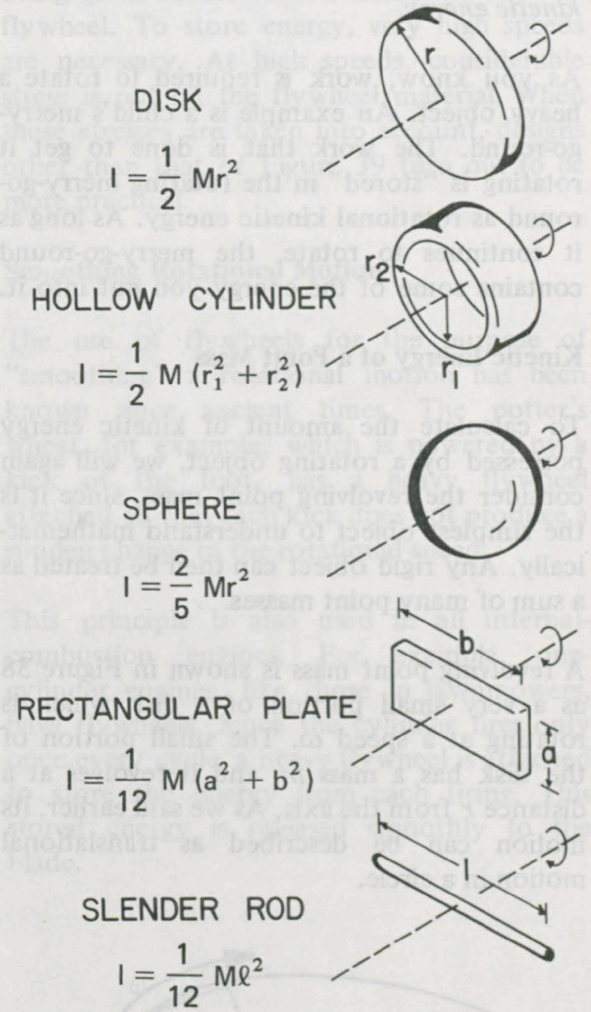


Figure 37. Equations for the moment of inertia of various simple rigid objects. In each case the rotation axis is indicated.

In each case, the rotation axis is indicated. If the object is rotated about a different axis, the moment of inertia is probably different. For example, what is the moment of inertia of the rod about an axis along its length?

ROTATIONAL KINETIC ENERGY

Another important aspect of the rotational motion of a rigid object is the amount of energy involved. The energy possessed by an object as a result of its motion is called *kinetic energy.* If the motion of the object is rotational, then this energy is called *rotational kinetic energy.*

As you know, work is required to rotate a heavy object. An example is a child's merry-go-round. The work that is done to get it rotating is "stored" in the rotating merry-go-round as rotational kinetic energy. As long as it continues to rotate, the merry-go-round contains some of the energy you put into it.

Kinetic Energy of a Point Mass

To calculate the amount of kinetic energy possessed by a rotating object, we will again consider the revolving point mass, since it is the simplest object to understand mathematically. Any rigid object can then be treated as a sum of many point masses.

A revolving point mass is shown in Figure 38 as a very small portion of a disk which is rotating at a speed ω. The small portion of the disk has a mass *m,* and it revolves at a distance *r* from the axis. As we said earlier, its motion can be described as translational motion in a circle.

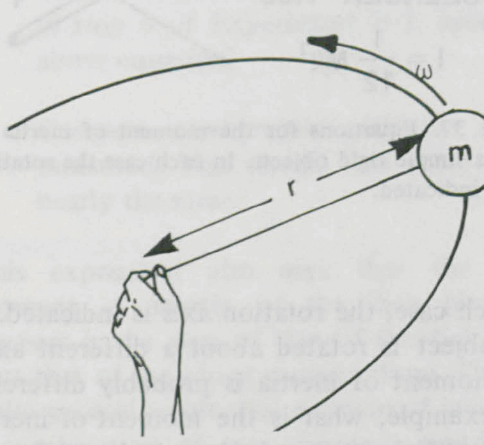


Figure 38. A small rotating mass shown as a small portion of a rotating disk.

The kinetic energy of a mass *m* traveling at speed *s* is:

$$KE = \tfrac{1}{2} ms^2$$

If *s* is the translational speed of the small mass going around in a circle, then this expression can be used to compute its kinetic energy. However, the translational speed of the point mass is related to the rotational speed of the disk by $s = r\omega$. Substituting this into the expression for kinetic energy gives the *rotational kinetic energy of the small revolving mass:*

$$KE = \tfrac{1}{2} mr^2 \omega^2$$

Note that, for a point mass, the translational kinetic energy and the rotational kinetic energy are just different ways of expressing the same energy.

Kinetic Energy of a Rotating Rigid Object

The total kinetic energy of a rotating rigid object is just the sum of the kinetic energies of all of its parts. Since all of its parts are rotating at the same rotational speed ω, then the sum of all of the $\tfrac{1}{2} mr^2 \omega^2$ is just the sum of the mr^2 times $\tfrac{1}{2} \omega^2$.

But the sum of the mr^2 is *defined* as the moment of inertia of the object *I.* Thus the *rotational kinetic energy of a rigid object* is:

$$KE = \tfrac{1}{2} I\omega^2$$

If *I* has units of kg·m and ω is in rad/s, then the kinetic energy is in *joules* (J).

This expression is very similar to that for translational kinetic energy ($\tfrac{1}{2} ms^2$). You simply replace translational speed *s* by rotational speed ω, and the translational inertia of the object *m* by its rotational inertia *I.* You must remember that *I* always depends on the axis about which the object rotates.

FLYWHEELS

An important use of rotational kinetic energy is in *flywheels.* A flywheel is a heavy rotating object which, by virtue of its rotational motion, has a lot of kinetic energy. Flywheels are generally used for two technological purposes: to store energy and to reduce the unevenness of a rotating system which only gets a power stroke every so often.

In either case, one usually wishes to maximize the rotational energy stored by the flywheel. The *KE* of a flywheel is proportional to the *square* of its rotational speed. Thus the faster the flywheel rotates, the greater its rotational kinetic energy. However, if it rotates too fast, it may break apart as a result of internal forces caused by the rotation.

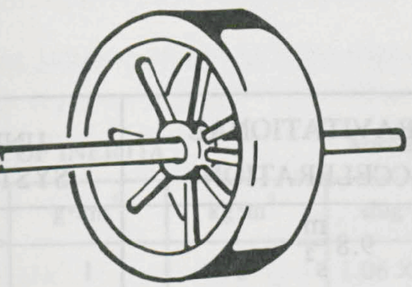


Figure 39. A common flywheel design for low speeds.

Since the rotational kinetic energy also depends on the moment of inertia of the object, the flywheel designer tries to obtain the largest possible I without making the wheel too heavy or too big. Look at the equations for I for various shapes on page 35. Note that all contain the expression Mr^2, but many are preceded by a reducing fraction. The ring shape, however, has no such fraction. Thus flywheels are frequently made in the shape of a heavy ring with a light support structure.

Storing Energy

Perhaps you've never thought about it, but energy is a particularly difficult thing to store. For example, assume for a moment that electric power is 50% cheaper at night than during the day. Can you think of a simple and inexpensive scheme to store energy during the night so you could use it during the day?

Your first thought might be to use a conventional storage battery. But they are expensive and not all that efficient. One way that is now being given considerable attention is to use a flywheel. To store energy, very high speeds are necessary. At high speeds, considerable stress is put on the flywheel material. When these stresses are taken into account, designs other than that of Figure 39 turn out to be more practical.

Smoothing Rotational Motion

The use of flywheels for the purpose of "smoothing" a rotational motion has been known since ancient times. The potter's wheel, for example, which is powered by a kick of the foot, has a heavy flywheel attached so that each kick does not produce a sudden change in the rotational speed.

This principle is also used in all internal-combustion engines. For example, one-cylinder engines, like those in lawnmowers, have flywheels. Since the cylinder fires only once every cycle, a heavy flywheel is attached to store the energy from each firing. This stored energy is released smoothly to the blade.

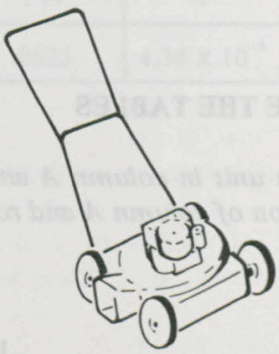


Figure 40. A lawn mower is an example of a one-cylinder engine.

A NOTE ABOUT UNITS

Various units and conversion factors for torque and moment of inertia are given on the next page. These require some explanation, since the units commonly used in the technical literature (which we call *engineering units*) are not always the proper units to use for calculations. The latter we will call *absolute units.*

The confusion results from the difference between force and mass. The table below gives the proper force and mass units for the various unit systems.

The conversion tables list both engineering *and* absolute units. *Any engineering unit must be converted to an absolute unit before making calculations.*

Torque

Torque has units of *force* times length. Thus the proper absolute units are dyne-centimeter, newton-meter, and pound-foot (or inch-ounce). However, commonly used engineering units are g•cm and kg•m since g and kg are often used as units of weight.

Moment of Inertia

Moment of inertia has units of *mass* times length squared. Thus the proper absolute units are: $g\cdot cm^2$, $kg\cdot m^2$, and $slug\cdot ft^2$. However, commonly used engineering units are $lb\cdot ft^2$ (or $oz\cdot in^2$), since we rarely use the English unit for mass, the slug.

WEIGHT	=	MASS	×	GRAVITATIONAL ACCELERATION	UNIT SYSTEM
newtons (N)	=	kilograms (kg)	×	$9.8\,\frac{m}{s^2}$	mks
dynes (dyn)	=	grams (g)	×	$980\,\frac{cm}{s^2}$	cgs
pounds (lb)	=	slugs	×	$32.2\,\frac{ft}{s^2}$	English

HOW TO USE THE TABLES

If you have a unit in column A and you want a unit in row B, multiply by the factor shown at the intersection of column A and row B.

For example:

$$1\ g\cdot cm^2 = 3.42 \times 10^{-4}\ lb\cdot in^2$$

TORQUE **Absolute Units** **Engineering Units**

A \ B	dyn·cm	N·m	lb·in	lb·ft	oz·in	g·cm	kg·m
dyn·cm	1	10^{-7}	8.85×10^{-7}	7.38×10^{-8}	1.42×10^{-5}	1.02×10^{-3}	1.02×10^{-8}
N·m	10^7	1	8.85	.738	142	1.02×10^4	.102
lb·in	1.13×10^6	.113	1	.0833	16	1150	.0115
lb·ft	1.36×10^7	1.36	12	1	192	1.38×10^4	.138
oz·in	7.06×10^4	7.06×10^{-3}	.0625	5.21×10^{-3}	1	71.9	7.19×10^{-4}
g·cm	980	9.80×10^{-5}	8.67×10^{-4}	7.23×10^{-5}	.0139	1	10^{-5}
kg·m	9.80×10^7	9.80	86.7	7.23	1390	10^5	1

MOMENT OF INERTIA **Absolute Units** **Engineering Units**

A \ B	g·cm²	kg·m²	slug·in²	slug·ft²	lb·in²	lb·ft²	oz·in²
g·cm²	1	10^{-7}	1.06×10^{-5}	7.38×10^{-8}	3.42×10^{-4}	2.37×10^{-6}	5.46×10^{-3}
kg·m²	10^7	1	106	.738	3420	23.7	5.46×10^4
slug·in²	9.43×10^4	9.43×10^{-3}	1	6.94×10^{-3}	32.2	.224	515
slug·ft²	1.36×10^7	1.36	144	1	4640	32.2	7.41×10^4
lb·in²	2930	2.93×10^{-4}	.0311	2.16×10^{-4}	1	.00695	16
lb·ft²	4.21×10^5	.0421	4.47	.0311	144	1	2304
oz·in²	183	1.83×10^{-5}	1.93×10^{-3}	1.35×10^{-5}	.0625	4.34×10^{-4}	1

SUMMARY

Section B has treated the factors that affect rotational motion. These factors include the cause of changes in rotational speed and the characteristics of the objects that rotate.

Changes in rotational speed are called *rotational accelerations.* They result from the application of external *torques.* Torque τ is defined as the applied force F times the perpendicular distance r_P from the rotation axis to the line along which the force is applied.

$$\tau = Fr_P$$

Stall torque is the torque that just prevents a motor from rotating. *Dynamic torque* is the torque exerted by the motor when it is rotating. The two motor torques are usually not the same for a given motor.

The equation describing changes in rotational speed is called the *equation of rotational motion.*

$$\tau = I\alpha$$

It relates the torque τ applied to a rigid object whose moment of inertia is I, to the amount of rotational acceleration α that it will produce. It explains rotational motion in the same way that Newton's second law, $F = ma$, explains translational motion.

Moment of inertia is the basic rotational property of an object. It depends on the body's mass and how that mass is distributed. Since the moment of inertia may be different for different axes of rotation of the object, the rotation axis must always be specified.

The moment of inertia of a single "point mass" m at distance r from the axis is:

$$I = mr^2$$

For dumbbell shapes and rings, where the total mass M is all at the same radius, the moment of inertia is:

$$I = Mr^2$$

For other simple shapes there are other formulas.

For more complicated rigid bodies, I can be measured. This is done by measuring the rotational acceleration produced by a known torque.

The *rotational kinetic energy* of a small part of mass m of a rotator is given by:

$$KE = \tfrac{1}{2}\, mr^2\,\omega^2$$

where m is the mass of that small part and r is the distance from m to the axis of rotation.

The kinetic energy of any rotating rigid body of moment of inertia I is:

$$KE = \tfrac{1}{2}\, I\omega^2$$

QUESTIONS

1. In "Soap Box Derby" races, unpowered cars roll down an incline. In going down the hill, the force of gravity does work on the cars, and they receive an amount of kinetic energy determined by the change in altitude. This kinetic energy is shared between the car's linear motion and the rotational motion of the wheels. How should the wheels be designed to give the highest possible final speed?
2. Explain, in terms of work and energy, what must be done to stop a moving car?
3. Explain briefly the meaning of each of the three quantities in the basic equation of rotational motion:

$$\tau = I\alpha$$

4. Why are record player turntables made of heavy metal rather than light plastic?

PROBLEMS

1. The stall torque of a small motor (Hurst model CA) is given by the manufacturer as 100 in·oz.

   a. Calculate the torque in lb·ft.

   b. The motor is connected to a pulley of 1-in radius, and a string is wrapped tightly around this pulley. How much weight could be lifted or supported by this motor?

2. The Sigma model 9AK4J2 stepping motor is advertized as having a hold torque of 950 g·cm. Since grams are units of mass, not force, what is meant is the *weight* of one gram times one centimeter. Convert this spec to dyn·cm.

3. An engine flywheel has almost all of its mass concentrated in a ring at an average distance of 15 cm (0.15 m) from the axis of rotation. Its mass is 9 kg. Calculate I in $kg{\cdot}m^2$.

4. A child's playground seesaw consists of a thin, narrow plank, 12 ft long and pivoted at its center. The mass of the plank is 18 kg. A child sits at each end of the plank. Assume each child has a mass of 45 kg.

   a. Calculate I for the plank alone about the axis at the center.

   b. Calculate I for each child about the same axis.

   c. Calculate the total I for the system.

   d. State briefly how the children could reduce I and still teeter.

5. A steady torque of 15 N·m acts on a wheel (free to turn) for 40 s. The wheel starts at rest and is turning at 1800 rpm after 40 s. Calculate the *KE* of the wheel after 40 s. (You need to find I first, from the information given.)

6. A grinding wheel consists of a flat disk 15 cm in diameter, with a mass of 500 g. It is driven by a motor that exerts a constant torque of 1.3×10^6 dyn·cm. How much time is required to bring the wheel up to 1725 rpm?

7. Calculate the dynamic torque of your fan motor for various supply voltages. Use the value of I from Experiment B-3 for the disk with all 24 of the washers attached and the values of acceleration from Experiment A-5. How does the dynamic torque compare with the stall torque of the motor, determined in Experiment B-1, for the same supply voltage?

Section B has treated the factors that affect [illegible] the causes of changes in rotational speed and the [illegible]

Changes in rotational speed are called rotational acceleration. [illegible]

[illegible] of rotational acceleration is that it will produce. It explains rotational motion in the same way that Newton's second law, $F = ma$, explains translational motion.

Moment of inertia is the basic rotational property of an object. It depends on the body's mass and how that mass is distributed. Since the moment of inertia may be different for different axes of rotation of the object, the rotation axis must always be specified.

The moment of inertia of a single "point mass" m at distance r from the axis is:

$$I = mr^2$$

For dumbbell shapes and rings, where the total mass M is all at the [illegible] moment of inertia is:

[illegible]

For other simple shapes, [illegible] formulas.

[illegible] measured. [illegible] rotational acceleration [illegible]

The *rotational kinetic energy* of a [illegible] of mass m of a rotator is given [illegible]

[illegible]

where m is the mass of that small [illegible] the distance from m to the axis [illegible]

The kinetic energy [illegible] of moment of inertia I [illegible]

[illegible]

QUESTIONS

1. In the "Soap Box Derby," [illegible] cars roll down an incline [illegible] of kinetic energy determined by the change in altitude. This kinetic energy is shared between [illegible] and the rotational motion of the wheels. How should the wheels be designed to give the highest possible final speed?

2. Explain, in terms of work and energy, what must be done to stop a moving car.

3. Explain briefly the meaning of each of the three quantities in the basic equation of rotational motion:

$$\tau = I\alpha$$

4. Why are record player turntables made of heavy metal rather than light plastic?

SECTION C

Rotational Balance

WHAT IS ROTATIONAL BALANCE?

You probably have a good idea of what the word "balance" means in normal situations. Applied to people, it means the ability to stay upright under varying circumstances. When weighing, it means an equilibrium between a known and an unknown mass. What other example of balance can you think of?

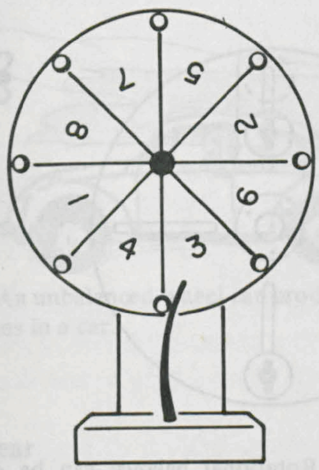


Figure 41. When a balanced wheel spins "where she stops nobody knows."

One definition of balance is *a stability produced by a proper distribution of mass.* This definition applies also to rotational balance. "Stability" for a rotating object is when the only rotational motion of the object is around the intended axis. That is, there are no other motions–for example, vibrations or wobbles–and there are no forces acting on the axle which would tend to produce such motions.

The "proper distribution of mass" which produces rotational stability is more difficult to explain, and often quite difficult to achieve. For one thing, there are two distinct kinds of rotational balance, *static balance* and *dynamic balance.*

Static Balance

Static balance means that when an object is stationary (static) and free to turn, it has no tendency to start rotating, regardless of its orientation. Thus to determine if a wheel is in static balance you can just turn it slightly and see if it rotates back with the same point down. That is, does it have a "heavy" spot? Having a heavy spot means that there is an unequal distribution of mass in the object.

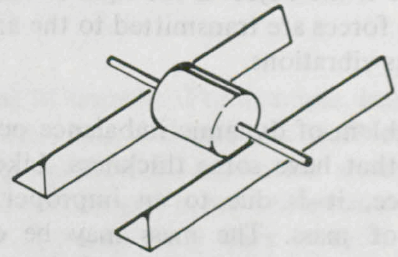


Figure 42. If the rotor has no tendency to roll, it is statically balanced.

This same principle is used in a slightly different, and more sensitive, way for automobile wheels. Static wheel balancing machines support the wheel *horizontally* by a needle at the center of its rotation axis. The wheel is then free to tilt. When it is level, it is statically balanced. You will use this method to statically balance the disk.

Figure 43. When the wheel is level it is statically balanced.

Dynamic Balance

The word "dynamic" means motion. Thus dynamic balance refers to a condition of an object when it is rotating. More specifically, a rotating object is dynamically balanced if it has no tendency to wobble as it turns.

If an object is out of static balance it will have a tendency to *vibrate* when it rotates. That is, the axle will tend to move back and forth slightly. If the object is statically balanced but out of dynamic balance, then it will tend to *wobble* on its axis.

A "wobble" is different from a vibration. A wobbling object tries to twist so that it can rotate around an axis that is different from the axle. If the object is too rigid to twist, the twisting forces are transmitted to the axle and appear as vibrations.

The problem of dynamic imbalance occurs in objects that have some thickness. Like static imbalance, it is due to an improper distribution of mass. The mass may be equally distributed on opposite sides of the axis but unequally distributed *along* the axis. Thus for objects that have both diameter and thickness, both static and dynamic balance are important.

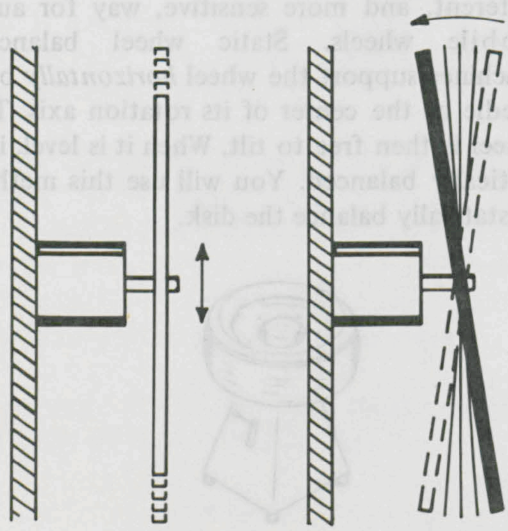


Figure 44. A disk out of static balance will tend to vibrate. A disk out of dynamic balance will tend to wobble.

Your Experiments

In the experiments of Section C you will observe the effects that an unbalanced disk has on the electric fan (if you haven't already done so). You will see how a statically unbalanced disk vibrates and a dynamically unbalanced disk wobbles. You will also see how these vibrations are transmitted to the entire fan and to whatever the fan touches.

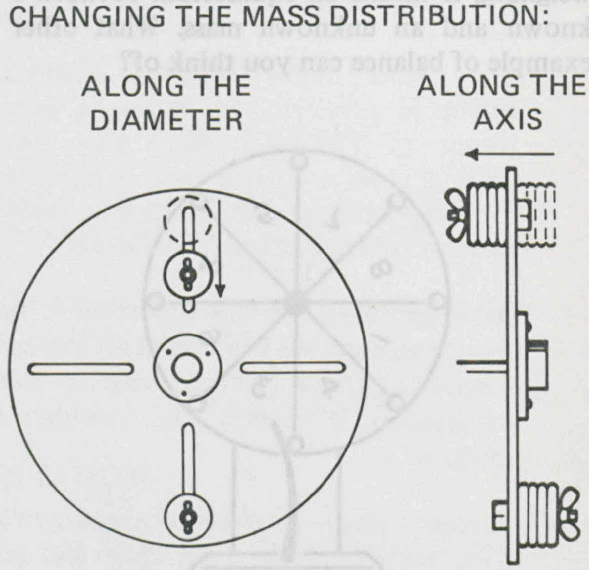


Figure 45. Rotational balance can be observed by changing the distribution of mass on the disk.

Then you will learn some of the techniques used to achieve static and dynamic balance. While it is fairly easy to statically balance the disk, dynamic balance requires a trial-and-error procedure that is far from exact.

Later in the module, we will discuss the forces and the torques that produce the vibrations and wobbles. We will also explain the reasons the balancing techniques worked.

On the following pages are some common examples of effects that are produced by rotating objects that are unbalanced. You have probably experienced some of these, and perhaps others as well.

THE EFFECTS OF IMBALANCE

Vibration

Rotational imbalance can cause serious destructive effects. The motion of an unbalanced rotor, such as the wheel of an automobile, causes forces that make it vibrate. These vibrations are transmitted to whatever is connected to the object. Not only can the results be annoying, but often the vibrations can cause serious damage to the structure.

Figure 46. An unbalanced wheel can produce destructive vibrations in a car.

Bearing Wear

The axles of rotating objects are usually supported by some type of bearing. These support bearings are designed to carry the weight of the rotating object, keep the shaft properly aligned, and produce very little friction. Bearings will last a long time if they are properly lubricated and the rotating object is properly balanced. When an imbalance occurs, forces are developed which rapidly wear away the bearing material.

Noise

Noise is another possible result of vibrations because sound is produced by vibrating objects. We know that excessive noise can be harmful to humans. For example, hearing can be permanently damaged and work efficiency severely reduced in high-noise situations. A noisy environment is an unpleasant place to be, and persons who work in such environments, even where the noise level is too low to cause hearing loss, suffer from increased tension.

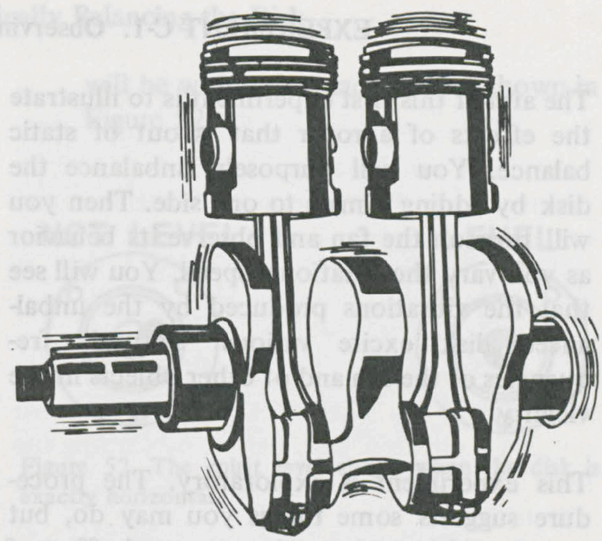


Figure 47. Forces from an improperly balanced crankshaft can quickly destroy the main engine bearing.

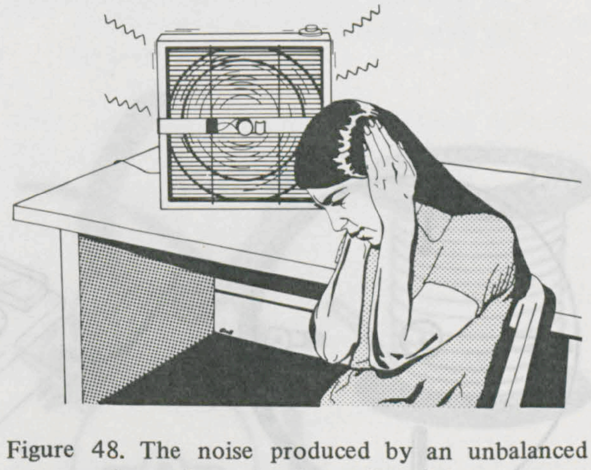


Figure 48. The noise produced by an unbalanced rotor can be unpleasant and even harmful.

EXPERIMENT C-1. Observing the Effects of Static Imbalance

The aim of this first experiment is to illustrate the effects of a rotor that is out of static balance. You will purposely unbalance the disk by adding a mass to one side. Then you will turn on the fan and observe its behavior as you vary the rotational speed. You will see that the vibrations produced by the unbalanced disk excite various "natural" frequencies of the fan and of other objects in the vicinity.

This experiment is exploratory. The procedure suggests some things you may do, but you should investigate the cause and effect of imbalance to your own satisfaction. However, remember that the vibrations you produce may be quite severe, so do not try to shake apart your laboratory.

Procedure

1. *Set up the arrangement shown below* with no extra weights on the disk. Turn the disk slightly and release it. Does it have any heavy spots?

2. *Turn on the fan* and observe its vibrations. Vary the rotational speed and see if you can excite any natural frequencies of its motion. This is similar to your experiment with the vibrating reed of Section A except that you are observing vibrations of the entire fan itself.

3. *Mount a bolt on one side of the disk* at the smallest radius possible. This produces a heavy spot so that the disk's "natural" rest position is with this bolt at the bottom.

4. *Turn on the power supply* and turn up the voltage until the disk starts to turn. Gradually increase the speed and observe the vibrations that occur. You should find that different speeds produce large vibrations of the fan in different directions. It also can cause other objects on the table to vibrate. Can you make the fan start to "walk"?

   CAUTION: *Do not let the fan shake too violently or too long. It is possible to cause damage.*

5. *Describe three different vibrations* you observe. It may help to use sketches.

6. *Explore the effects* of adding washers at various positions.

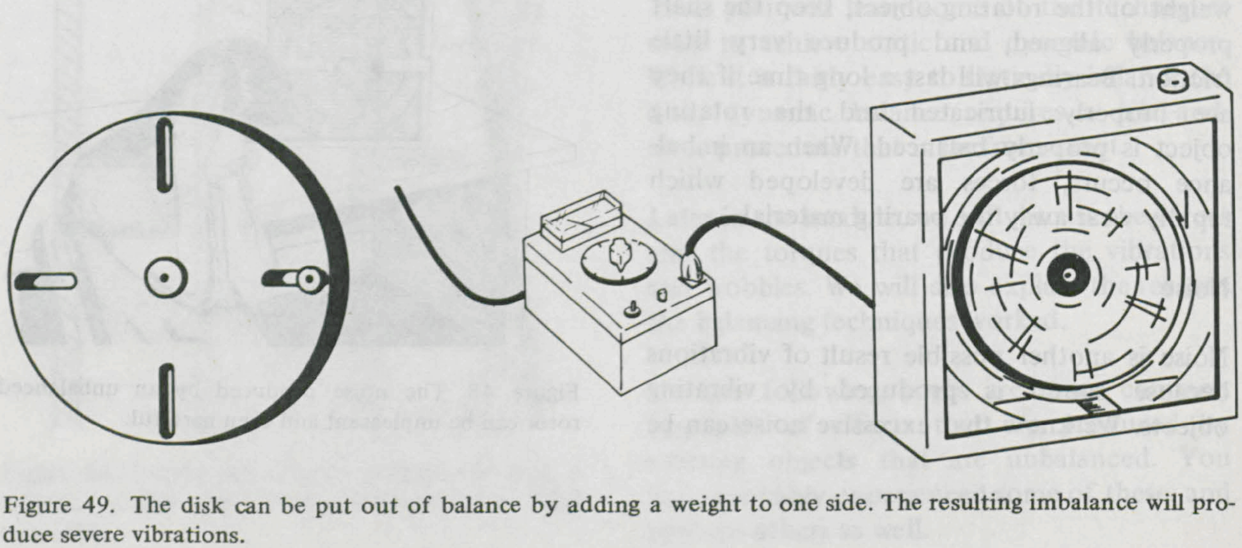


Figure 49. The disk can be put out of balance by adding a weight to one side. The resulting imbalance will produce severe vibrations.

EXPERIMENT C-2: Statically Balancing the Disk

In this experiment you will carefully balance the disk statically to see if you can minimize the vibrations. The static-balancing technique is quite sensitive, and is the same one used to statically balance automobile wheels. Here you will use a simple device used for statically balancing lawn mower blades, and heavy solder to achieve the proper distribution of mass. In automobile wheel balancing, specially designed lead weights are used.

Procedure

1. *Remove the disk from the fan* and put four washers on one bolt and three on another in the opposite slot. Initially, locate both bolts at the maximum radius.

Figure 50. Unbalance the disk by adding unequal masses.

2. *Place the disk on the static balancer,* as shown in cross section in Figure 51. The disk will tilt because it is unbalanced.

3. *Place the spirit level on the center of the disk.* When the disk is level, the bubble will be within the black ring, as shown in Figure 52.

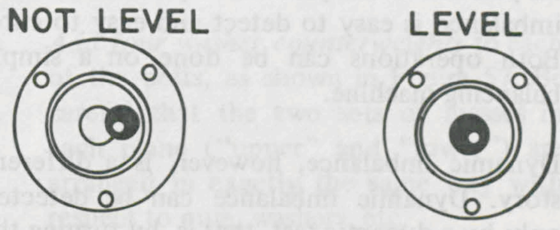


Figure 52. The spirit level shows when the disk is exactly horizontal.

4. *Coarsely balance the disk* by adjusting the position in the slot of the four washers. You probably cannot do it perfectly. This shows, however, that it is not mass alone that produces static balance; location is also important.

5. *Finely balance the disk* by adding a small piece of solder. Cut and clamp a short length under the three outer washers with a short "pigtail" sticking out. Trim off small pieces of the pigtail until the disk becomes level.

6. *Rotate the disk on the fan* and see how well you did. Vary the speed and see if any vibrational reasonances occur.

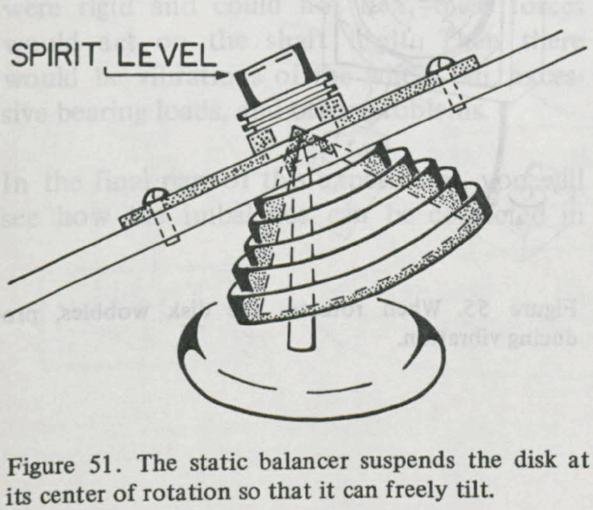


Figure 51. The static balancer suspends the disk at its center of rotation so that it can freely tilt.

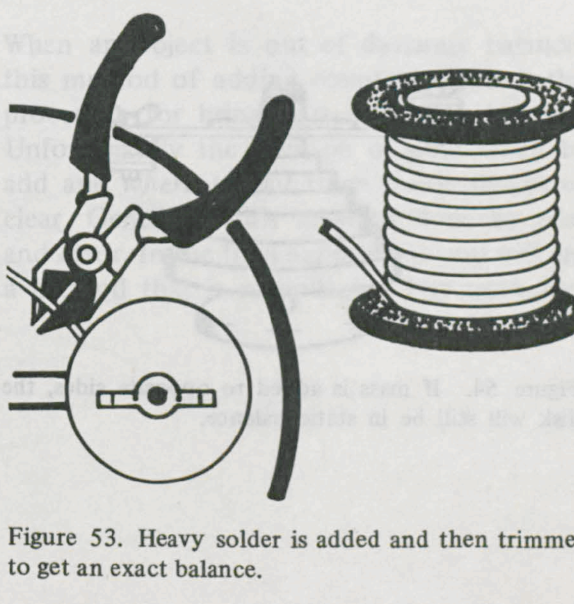


Figure 53. Heavy solder is added and then trimmed to get an exact balance.

EXPERIMENT C-3: Observing the Effects of Dynamic Imbalance

Dynamic balance will be discussed fully in the remainder of the text, but now the aim is for you to get some practical experience with it. As you saw in the previous experiments, static imbalance is easy to detect and easy to cure. Both operations can be done on a simple balancing machine.

Dynamic imbalance, however, is a different story. Dynamic imbalance can be detected only by a dynamic test, that is, by turning the rotor at high speeds and looking for certain effects.

Dynamic imbalance can cause severe vibration, but the effect you will look for here is a slightly different one which we shall call *wobble.* The experiment should help clarify the meaning of wobble. In brief, it means a motion where the plane of the rotor no longer stays perpendicular to the axis of rotation.

Procedure

1. *Place two identical sets of four washers on opposite slots and on opposite sides of the disk.* This change in relative positions of the masses is a key feature of dynamic imbalance. The masses are not in the same plane perpendicular to the axle and the mass distribution along the axle is no longer uniform.

2. *Statically balance the disk.* Adjust the position of one set of washers to achieve a coarse static balance. Then finely balance the disk by adding solder, as described in Experiment C-2. Note that the reversal of sides does not affect your ability to get a static balance.

3. *Mount the disk on the fan shaft.*

4. *Turn on the fan* at a low voltage so that the disk turns at a moderate speed. Observe the behavior at various speeds. Can you detect any wobble?

5. *"Stop" the rotation with a strobe.* First stop the motion at the *highest* flash rate that gives a *single* image of a reference mark on the rotor. Then change the flash rate slightly so that the rotor appears to rotate slowly. You will then see the wobble in the "slow motion."

6. *Measure the size of the wobble (amplitude)* by doubling the flash rate of the

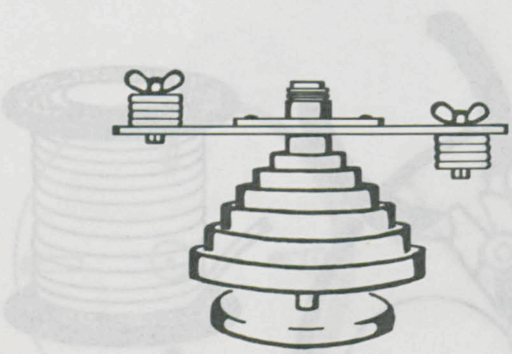


Figure 54. If mass is added to opposite sides, the disk will still be in static balance.

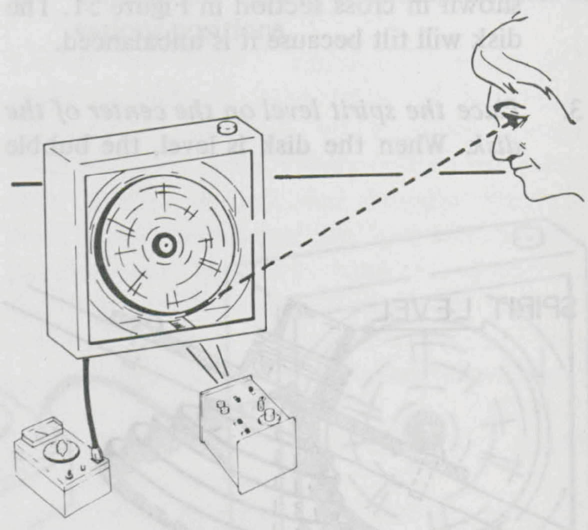


Figure 55. When rotated, the disk wobbles, producing vibration.

strobe. At a flash rate of twice the rpm, you will see the two extremes of the wobble. Read the amplitude along the cm scale on the frame by sighting over the edge of the disk. (See Figure 55.) You should see something like that shown below.

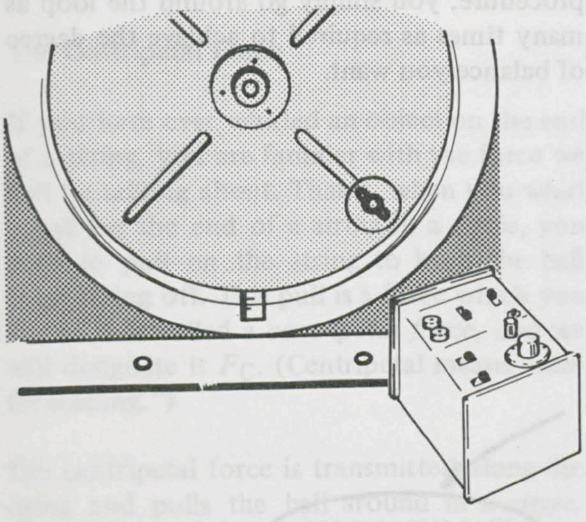


Figure 56. The amplitude of the wobble can be measured by stopping the motor with a strobe.

7. *Sketch a side view of the disk* as it rotates showing its angle with respect to the axle. Show also the positions of the washers.

You have just seen the result of dynamic imbalance: forces arise and produce a wobbling motion. The rotor no longer stays aligned with respect to the shaft. If the rotor were rigid and could not flex, these forces would act on the shaft itself. Then there would be vibrations of the whole fan, excessive bearing loads, and other problems.

In the final part of this experiment, you will see how the imbalance can be corrected in this simple special case. The key word is *counterweights:* masses added to keep the total mass evenly distributed in planes perpendicular to the axle. Of course, when mass is added, the static balance must be checked and preserved.

8. *Add four washer counterweights* to each of the bolts, as shown in Figure 57. Be careful that the two sets of masses in each plane ("upper" and "lower") are arranged in exactly the same way with respect to nuts, washers, etc.

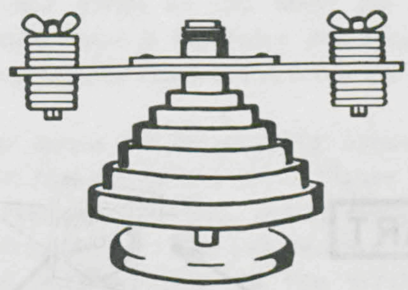


Figure 57. Adding counterweights will restore the dynamic balance.

9. *Statically balance the disk* in the usual way, using the static balancer. Keep the bolts near the maximum radius.

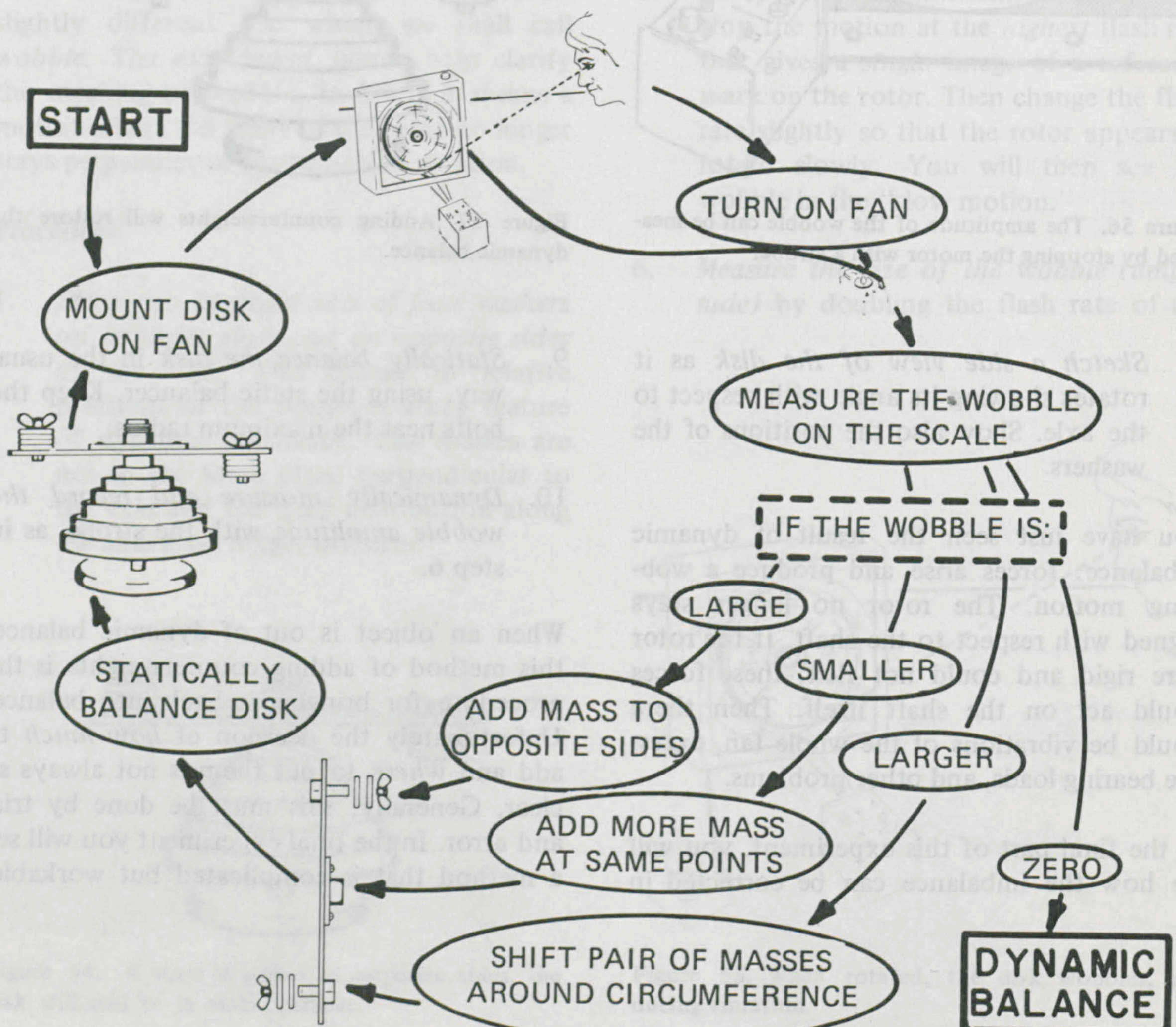
10. *Dynamically measure and record the wobble amplitude* with the strobe, as in step 6.

When an object is out of dynamic balance, this method of adding counterweights is the procedure for bringing it back into balance. Unfortunately the decision of *how much* to add and *where* to put them is not always so clear. Generally, this must be done by trial and error. In the final experiment you will see a method that is complicated but workable.

EXPERIMENT C-4: Dynamically Balancing a Disk

Dynamic imbalance is a common problem, particularly for thick or long rotors such as motor armatures or crankshafts. It is almost never obvious where or how much mass should be added to correct the problem. In this experiment, you will learn a relatively simple procedure for measuring and correcting dynamic imbalance of thin rotors. It is not too precise, but it will at least give you an idea of what is involved in dynamic balancing.

Your teacher will provide you with a symmetric, but *dynamically unbalanced,* disk. Your task is to add weights at the proper places to bring it into dynamic as well as static balance. The procedure is illustrated in the circular flow chart below. Since it is a trial-and-error procedure, you simply go around the loop as many times as required to achieve the degree of balance you want.



FORCES FROM ROTATION

The vibrations and wobbles that you observed in your experiments were the result of forces. The forces, in turn, were the direct result of the rotation. Therefore, let us begin an exploration of rotational balance by analyzing the forces produced by rotation.

The Centripetal Force

If you have ever whirled an object on the end of a string, you are familiar with the force we will be talking about. That is, when you whirl a ball on the end of a string in a circle, you have to pull on the string to keep the ball from flying off. This pull is a force which you exert. It is called a *centripetal force,* and we will designate it F_C. (Centripetal means "center seeking.")

The centripetal force is transmitted along the string and pulls the ball around in a circle. Since it is a pull on the ball, it can be represented by an arrow from the ball directed inward along a radius (the string) toward the center of rotation.

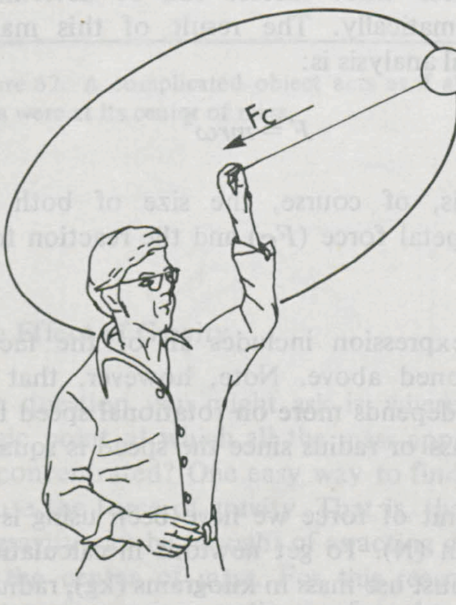


Figure 58. You must exert an inward (centripetal) force on the ball to keep it from flying off.

The Reaction Force

Newton's third law, the law of action and reaction, states that every force is accompanied by an equal but opposite *reaction force.* The reaction force always acts on a different object than does the so-called *action force.* For the whirling ball, this means that there is a reaction force equal to the centripetal force, but opposite in direction. This reaction force, which we shall call F_R, is directed outward, always along a radius of the circle.*

Since this reaction force is outward, it pulls on your hands as you whirl the ball. The reaction force is the force *you* feel, whereas the centripetal force is the force the *ball* feels.

These terms can be generally applied to any object that moves in a circle. Figure 59 shows, for example, the force acting on the bolt of Experiment C-1. The disk exerts a centripetal force on the bolt, and this keeps it from flying off. The bolt, in turn, exerts a reaction force on the axle. It is this kind of reaction force on the disk and axle which produces the vibrations and wobbles which you observed in your experiments.

*This outward force is frequently called a *centrifugal force.*

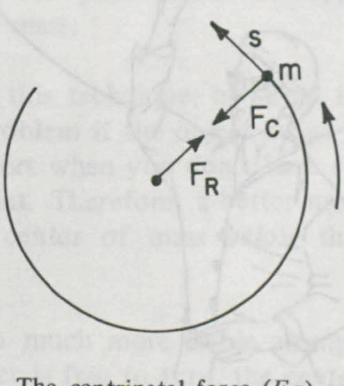


Figure 59. The centripetal force (F_C) on a whirling object is accompanied by an equal but opposite reaction force (F_R).

What Determines the Size of the Forces?

The fact that the reaction force exists is really all you need to know to understand how vibrations and wobbles result from imbalance. However, it is also possible to derive an expression for the size of this force. Since we know it is equal (but opposite) to the centripetal force, an expression for the centripetal force is also the expression for the reaction force.

The first question to ask is what factors would such an expression include? That is, what characteristics of a whirling object would you expect to determine the size of the force you have to exert to keep it going in a circular path? Conversely, what is the size of the force the object exerts on your hand as it whirls?

A Simple Experiment

Tie an object to a string and whirl it. As you do, try to decide what factors are determining the size of the force you exert. If you can't do this experiment just now, then do it mentally, thinking about what factors you believe would be important.

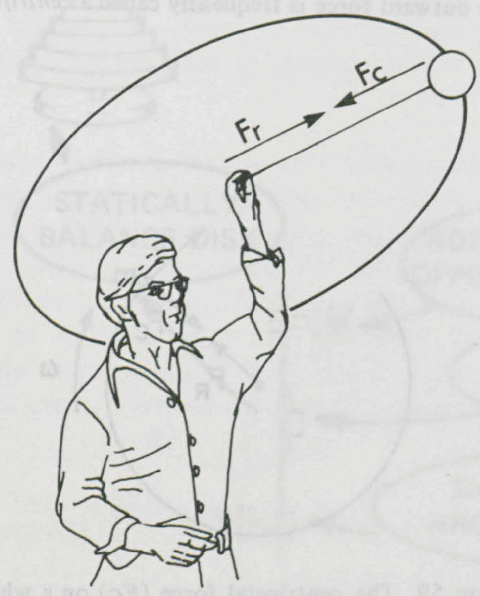


Figure 60. Whirl an object to determine what factors affect the centripetal force.

One of the factors involved is mass—the more massive the object is, the more force you have to exert. Another factor is rotational speed—the faster it goes, the more force you have to exert. The final factor is radius—the longer the string, the bigger the force you have to exert.

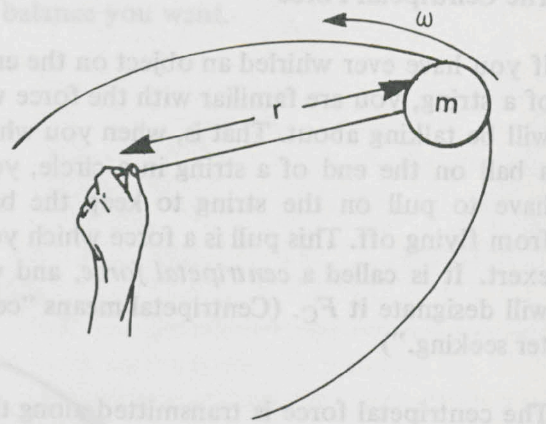


Figure 61. The key factors of centripetal force are mass, radius, and rotational speed.

The Mathematical Result

The exact dependence of the centripetal force on these three factors can be determined mathematically. The result of this mathematical analysis is:

$$F = mr\omega^2$$

This is, of course, the size of both the centripetal force (F_C) and the reaction force (F_R).

The expression includes all of the factors mentioned above. Note, however, that the force depends more on rotational speed than on mass or radius since the speed is squared.

The unit of force we have been using is the newton (N). To get newtons in calculations, you must use mass in kilograms (kg), radius in meters (m), and rotational acceleration in radians per second (rad/s).

CENTER OF MASS

There is one final idea needed before we can explain static and dynamic balance. That is the idea of *center of mass.* Under the influence of external forces, an object *acts* as if its entire mass were concentrated at a single point, called the center of mass.

For example, suppose you throw an object with a strange shape, such as a tennis racket. As it sails through the air, its motion appears quite complicated. But the motion of its center of mass is simple. It is identical to that of a baseball. The racket may spin around, but its center of mass follows the same smooth parabola as a baseball.

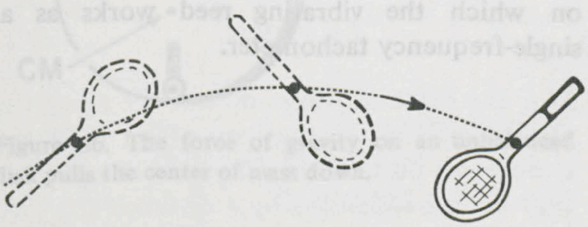


Figure 62. A complicated object acts as if all of its mass were at its center of mass.

The Effect of Gravity

One question you might ask is, where is the magic point at which all the mass appears to be concentrated? One easy way to find out is to use the force of gravity. That is, the force of gravity can be thought of as acting directly on the center of mass. For this reason, the center of mass is sometimes referred to as the *center of gravity.*

If the object has a support directly under the center of mass, then the force of gravity acts directly on the support and the object has no tendency to fall off. If the center of gravity is to the side of the support, however, then the force of gravity will pull it over. Gravity will exert a torque on the object. The torque about the point of support will be the product of the gravitational force and the distance between the center of gravity and the support. This is indicated in Figure 63.

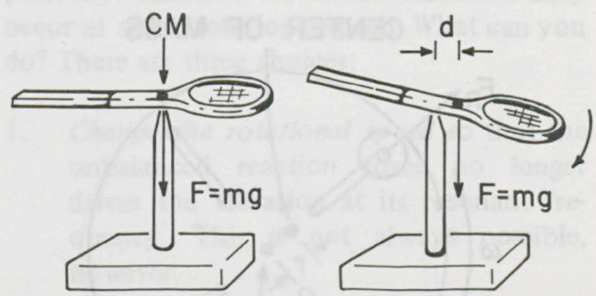


Figure 63. Gravity can help you locate the center of mass.

How Do You Find the Center of Mass?

The gravitational torque provides a straightforward way of determining where the center of mass of an object is. You just put a support under it and keep moving it around until it balances. The point above the support is the center of mass.

In using this technique, however, it is generally a problem if the object keeps falling off the support when you don't have *exactly* the right point. Therefore, a better method is to get the center of mass *below* the support point.

This is a much more stable arrangement. If the object is free to turn, the center of mass will automatically hang directly under the support point.

STATIC IMBALANCE

Its Cause

When an object rotates about an axis that does not pass through the center of mass, the object *acts* as if all its mass M were concentrated at the center of mass. This rotating center of mass produces a reaction force on the axis and thus on the bearings. It rotates at the rotational speed, pulling the bearings as it goes.

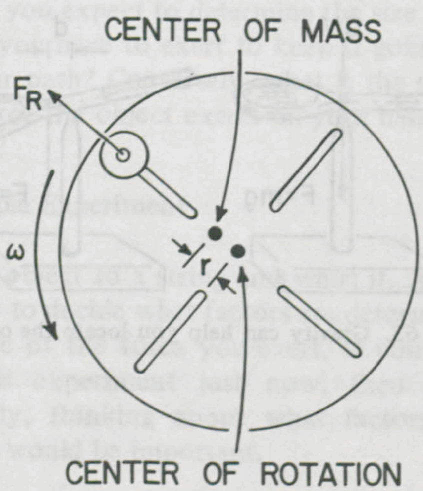


Figure 64. If the center of mass is not at the center of rotation, an unbalanced reaction force pulls on the bearing.

The size of this reaction force is fairly simple to calculate. The center of mass is separated from the axis of rotation by a distance r. The reaction force is the same as if the whole disk were reduced to a point mass rotating at a radius r. This would yield a *reaction force on the bearing of:*

$$F_R = Mr\omega^2$$

While r is typically quite small, sometimes only thousandths of an inch, M is the whole mass of the object. Thus F_R can get quite large, particularly for large rotational speeds ω.

How It Produces Vibration

A real system like the fan has a number of preferred modes in which it "likes" to vibrate. Each mode has a specific natural frequency, much as does the vibrating reed.

When the disk rotates, the unbalanced reaction force rotates around with it at the rotation frequency. This tends to pull the fan back and forth at the same frequency in every direction in the plane of the disk.

In other words, there is an oscillating driving force in each direction at the frequency of rotation. When this frequency happens to match a natural frequency of vibration, large vibrations are produced.

This condition is called *resonance.* Different rotational speeds excite different vibrational motions of the fan, as you observed in your experiment. This same principle is the basis on which the vibrating reed works as a single-frequency tachometer.

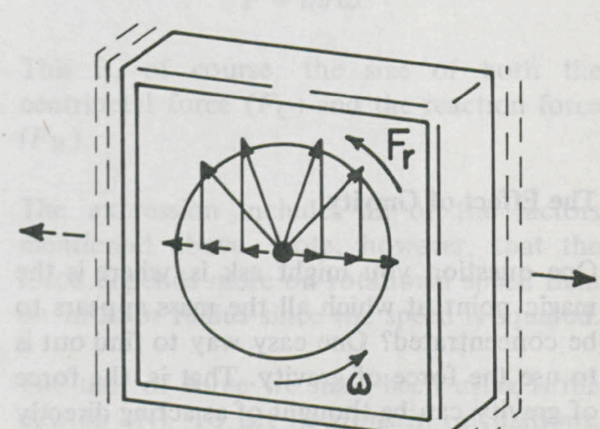


Figure 65. The reaction force F_R turns and drives any natural modes at the frequency of rotation.

The Cure for Static Imbalance

The cure for static imbalance is to be sure that the rotation axis of an object passes through its center of mass. While it is usually difficult to move the rotation axis, it is generally not difficult to move the center of mass. Simply add additional mass on the opposite side.

There are two ways to determine on which side of the rotor the center of mass may be. A rotor suspended *vertically* on its shaft can give a rough idea, since gravity pulls the heavy side down. This is only a rough indication, however, since most bearings have quite a bit of friction.

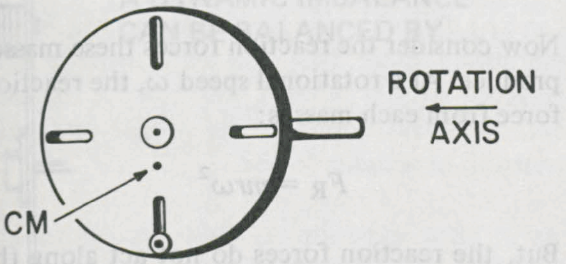


Figure 66. The force of gravity on an unbalanced disk pulls the center of mass down.

A considerably more sensitive method is to suspend the rotor *horizontally* at its axis by a needle bearing, as you did with the static balancing machine. Here, the center of mass needs to be below the point of suspension for stability.

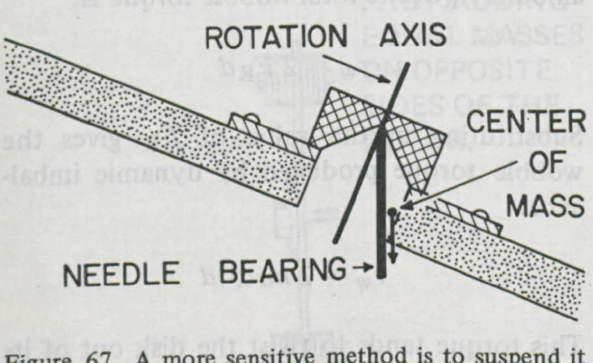


Figure 67. A more sensitive method is to suspend it horizontally at the center of rotation on a needle bearing.

Static balance is achieved by adding mass at the appropriate place to move the center of mass over to the rotation axis. Either method can be used to show when this has been achieved. In the method of Figure 66, there will be no tendency for the object to have a preferred side down. In the method of Figure 67, the object will remain level.

The Cure for Vibration

Sometimes it happens that you do not have access to the rotating object to balance it. For example, the rotor of the fan motor may be out of balance, even though the disk is perfectly balanced. A severe vibration may occur at some rotational speed. What can you do? There are three choices:

1. *Change the rotational speed* so that the unbalanced reaction force no longer drives the vibration at its resonant frequency. This is not always possible, however.

2. *Change the resonant frequency.* This can often be done by mounting the system on a heavy base.

3. *Damp the vibrations.* Mount the system on a base that absorbs the energy of vibration by friction. This is called *damping* the vibration.

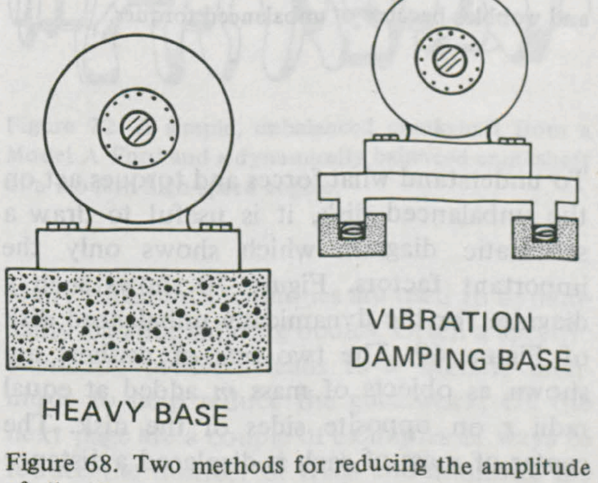


Figure 68. Two methods for reducing the amplitude of vibrations.

DYNAMIC IMBALANCE

Its Cause

To explain static imbalance we looked at the reaction forces present and how they acted on the axle. To explain dynamic imbalance, it is necessary to determine if these forces produce any torques.

In your experiments with the fan, the rotor was a thin disk, almost a single plane. When it rotated, its plane was perpendicular to the rotation axis. All reaction forces acted in the plane of the disk and were perpendicular to the axis. However, when you added washers to opposite sides of the disk, the analysis changed.

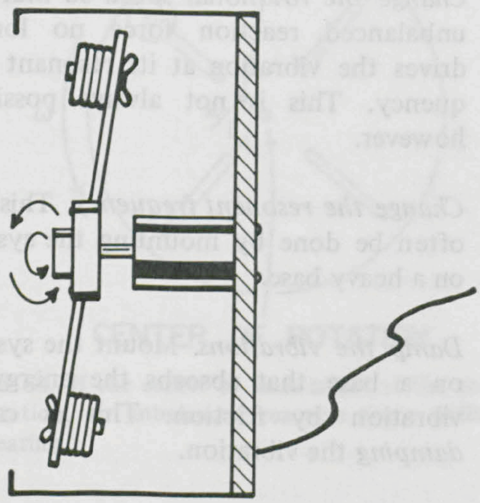


Figure 69. The dynamically unbalanced disk flexes and wobbles because of unbalanced torques.

To understand what forces and torques act on the unbalanced disk, it is useful to draw a schematic diagram which shows only the important factors. Figure 70 shows such a diagram for the dynamically unbalanced disk of Figure 69. The two sets of washers are shown as objects of mass m added at equal radii r on opposite sides of the disk. The center of mass of each is displaced a distance d from the plane of the disk.

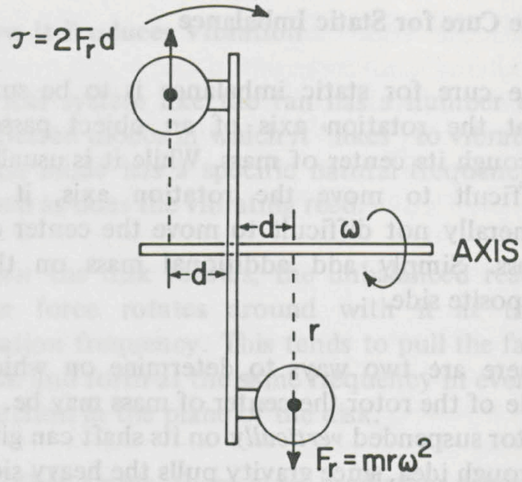


Figure 70. Schematic drawing of a dynamically unbalanced object.

How It Produces Wobble

Now consider the reaction forces these masses produce. At a rotational speed ω, the reaction force from each mass is:

$$F_R = mr\omega^2$$

But, the reaction forces do not act along the same line. That is, they are offset from the same point on the axis. The lines along which they act are separated by a distance $2d$. Two forces like these are called a *couple*.

Because of being offset, each force produces a torque about the point where the disk is attached to the axle. Each torque tends to bend the disk in a direction perpendicular to its plane. Since there are two such torques of amount $F_R d$, the total *wobble torque* is:

$$\tau_w = 2\,F_R d$$

Substituting in the value of F_R gives the wobble torque produced by dynamic imbalance:

$$\tau_w = 2\,mr\omega^2 d$$

This torque tends to twist the disk out of its normal position perpendicular to the rotation axis, and causes it to wobble as it turns.

The disk used in your experiments was flexible, so it bent to a tilted position as it turned. (Check your sketches of Experiment C-3.) If the disk were more rigid, then it could not bend. Instead, the wobble torque would be transmitted to the axle and bearings to produce strain, wear, and vibration.

The Cure for Dynamic Imbalance

To eliminate dynamic imbalance, one must either redistribute the mass of the rotor so that no wobble torques exist, *or* add torques that are equal and opposite to the ones that are present. If static balance already exists, a *couple* must be added.

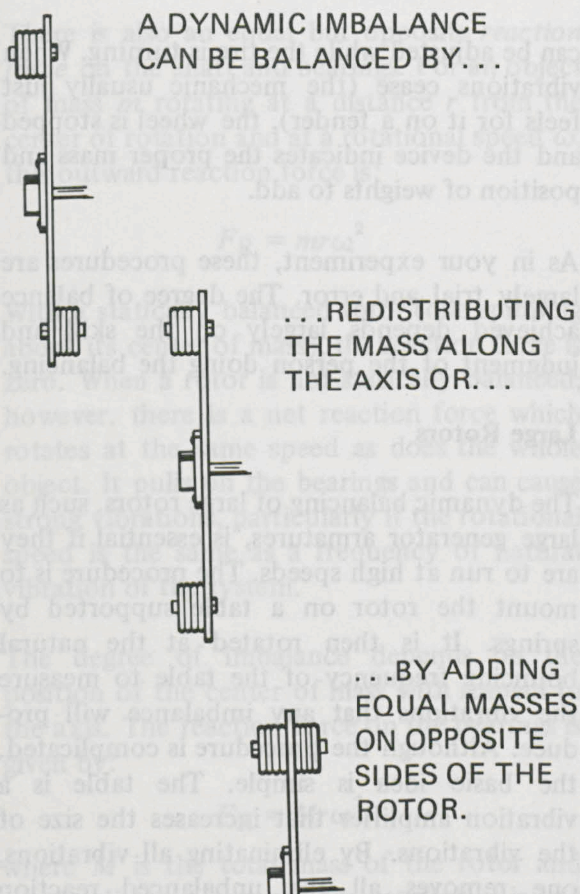


Figure 71. How to cure dynamic imbalance.

Adding a couple means to add a pair of weights (to preserve static balance) on opposite ends of a diameter, *and* on opposite sides of the rotor.

In your first experiments on dynamic imbalance, the solution was clearcut. Since the unbalanced masses were large, you could *see* where they were, and you could easily move them. In your last experiment, as with most rotating bodies, the unequal distribution of mass was neither known nor could it be moved. Thus a trial-and-error procedure was adopted to attempt to improve the problem.

An Example

Dynamic imbalance occurs most often in thick rotors–rotors that extend along the axis for some distance (*d* large). Examples are tires, motor armatures, and engine crankshafts. Imbalance effects increase with the square of the rotational speed, so crankshaft balance is of great importance in racing and high-performance engines.

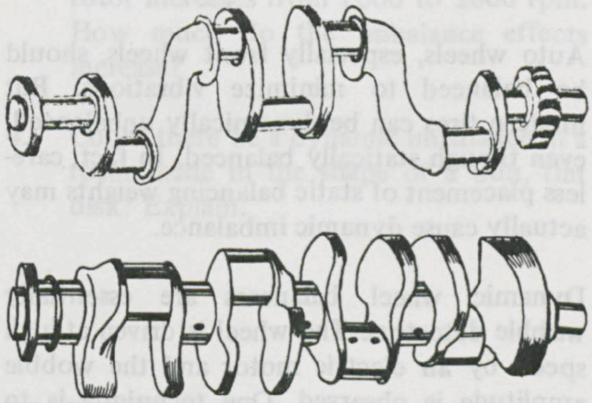


Figure 72. A simple, unbalanced crankshaft from a Model A Ford and a dynamically balanced crankshaft of a modern high-speed engine.

Many different techniques are used to dynamically balance rotating bodies. Often a specific balancing problem leads to a specific technique to help reduce the guesswork. On the next page are a couple of examples of ways to reduce the number of trials and minimize the resulting errors.

BALANCING TECHNIQUES

Automobile Wheels

Static balancers for automobile wheels are essentially the same as the device you used to statically balance the disk. They are easy to use and are usually effective.

The wheel is suspended horizontally at its center of rotation, and lead weights are added to the rim until a level indicates that the wheel is exactly horizontal.

Figure 73. Static balancer for an automobile wheel.

Auto wheels, especially front wheels, should be balanced to minimize vibrations. But modern tires can be dynamically unbalanced, even though statically balanced. In fact, careless placement of static balancing weights may actually cause dynamic imbalance.

Dynamic wheel balancers are essentially wobble detectors. The wheel is driven at high speed by an electric motor and the wobble amplitude is observed. One technique is to mount a small probe near the rim. As the wheel wobbles, the position of the probe, when the tire just strikes it, measures the wobble amplitude. The probe can be used to trigger a strobe which shows where masses should be added to the wheel.

A more common method is to attach a device to the rim of the wheel with which weights

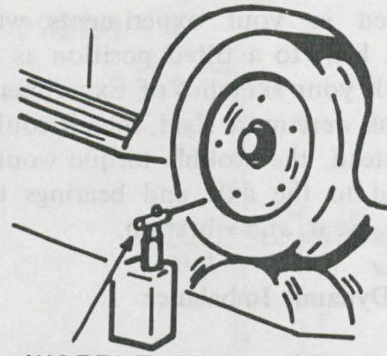


Figure 74. A method for dynamically balancing automobile wheels.

can be adjusted while the tire is turning. When vibrations cease (the mechanic usually just feels for it on a fender), the wheel is stopped and the device indicates the proper mass and position of weights to add.

As in your experiment, these procedures are largely trial and error. The degree of balance achieved depends largely on the skill and judgment of the person doing the balancing.

Large Rotors

The dynamic balancing of large rotors, such as large generator armatures, is essential if they are to run at high speeds. The procedure is to mount the rotor on a table supported by springs. It is then rotated at the natural bouncing frequency of the table to measure the vibrations that any imbalance will produce. Although the procedure is complicated, the basic idea is simple. The table is a vibration amplifier that increases the size of the vibrations. By eliminating all vibrations, one removes all the unbalanced reaction forces and therefore the torque wobble. This procedure results in both static *and* dynamic balance of the rotor.

SUMMARY

Static and dynamic balance of a rotating object are important, since imbalance can cause large forces acting on the axle. These forces lead to noise, vibration, and rapid wear of bearings and other moving parts.

Static balance means that an object, supported by its axle, has no tendency to turn. If the *center of mass* coincides with the rotation axis, the rotor may turn smoothly, without vibration.

Rotating objects are acted on by *centripetal forces,* which act inward. Centripetal forces produce the deflection necessary to keep the object traveling in a circular path.

There is also an equal but opposite *reaction force* on the shaft and bearings. For an object of mass m rotating at a distance r from the center of rotation and at a rotational speed ω, this outward reaction force is:

$$F_R = mr\omega^2$$

With a statically balanced rotor (one spinning about its center of mass), the reaction force is zero. When a rotor is not statically balanced, however, there is a net reaction force which rotates at the same speed as does the whole object. It pulls on the bearings and can cause strong vibrations, particularly if the rotational speed is the same as a frequency of natural vibration of the system.

The degree of imbalance depends on the position of the center of mass with respect to the axis. The reaction force on the bearings is given by:

$$F_R = Mr\omega^2$$

where M is the total mass of the rotor and r is the distance between the rotation axis and the center of mass.

Static imbalance is corrected by adding mass at an opposite position. Static balancing machines use a spirit level to indicate when the center of mass is at the rotation axis.

Dynamic balance means that an object has no tendency to wobble when rotating. Dynamic imbalance occurs when there is an unequal distribution of mass *along* the axis. The reaction forces of the distribution produce a torque. This *wobble torque* tends to make the rotor and the shaft wobble about an axis perpendicular to the rotation axis.

For rotors that are already statically balanced, dynamic imbalance is corrected by adding pairs of masses in two planes along the axis. Pairs of masses are needed to maintain static balance.

QUESTIONS

1. Suppose you cannot add mass to a statically unbalanced rotor, but were permitted to drill holes in it. What would you do to achieve balance?

2. State the important property of the center of mass.

3. The rotational speed of an unbalanced rotor increases from 1000 to 2000 rpm. How much do the imbalance effects increase?

4. Could there be a dynamic imbalance in a rotor made in the shape of a thin, flat disk? Explain.

PROBLEMS

1. A rotor has a mass of 10 kg; its center of mass is 1 cm from the axis. Where should a 0.1-kg weight be placed to statically balance it? (Hint: $mr\omega^2$ for the added weight should be equal to $Mr\omega^2$ for the center of mass. Why?)

2. A rotor consists of a very long but very light rod with a 2-kg mass attached to one end. It is to rotate about an axis 10 cm from the mass, as shown in Figure

75. If you have a 0.2-kg mass which can be attached to the rod, how would you balance it?

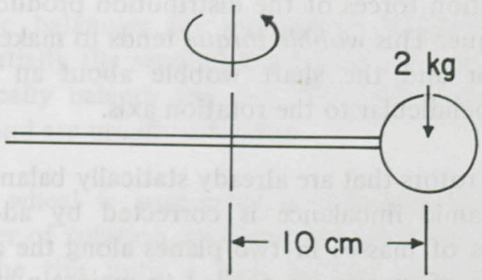


Figure 75.

3. A rotor has a total mass of 5 kg and its center of mass is 1 cm from the axis. What is the reaction force on the bearings at 3000 rev/min?

4. The bearings on a certain rotor can withstand safely a reaction force of 100 N (about 20 lb). The rotor has a mass of 10 kg and the center of mass is 1 mm from the axis. How fast can the rotor turn?

5. Dynamic imbalance results from two 100-g masses at a radius of 30 cm. Their separation along the axis is 2 cm. Calculate the wobble torque at 300 rad/s.

DATA PAGE

EXPERIMENT A-3. Calibrating a Tach Generator

Tach Voltage (V)	Strobe Rate (fpm)

Tach Generator Output Rating

Tach generator output = ____________ V

Rotational speed = ______ × 1000 rpm

$$R = \frac{\text{tach generator output}}{\text{rotational speed}}$$

$$= \frac{\underline{\qquad\qquad}\ \text{V}}{1000\ \text{rpm}}$$

= ____________ V/(1000 rpm)

EXPERIMENT A-5. Measuring Rotational Acceleration

Voltage ____________ V Voltage ____________ V

Voltage ____________ V Voltage ____________ V

COMPUTATION SHEET

EXPERIMENT B-1. Measuring Torque

Mass Hanging (kg)	R (m)	Weight Mg (N)	Stalling Torque FR_P (N·m)

EXPERIMENT B-2. Torque and Rotational Acceleration

Radius of Hub __________ m

A. Mass _________ kg Force _________ N.

Torque __________ N·m

Time (s)	Final Rotational Speed (rad/s)

Average Δt = _______ s $\Delta\omega$ = _______ rad/s

B. Mass _________ kg Force _________ N.

Torque __________ N·m

Time (s)	Final Rotational Speed (rad/s)

Average Δt = _______ s $\Delta\omega$ = _______ rad/s

C. Mass ________ kg Force ________ N.

Torque ________ N·m

Time (s)	Final Rotational Speed (rad/s)

Average Δt = ______ s $\Delta\omega$ = ______ rad/s

D. Mass ________ kg Force ________ N.

Torque ________ N·m

Time (s)	Final Rotational Speed (rad/s)

Average Δt = ______ s $\Delta\omega$ = ______ rad/s

Frictional torque τ_0 = __________ N·m

EXPERIMENT B-3. Rotational Characteristics of Rigid Objects

Mass Attached	Time (s)	Final Rotational Speed (rad/s)	Rotational Acceleration (rad/s^2)

Radius of attachment ________________ m.

Mass of one bolt ________________ kg.

Mass of one washer ________________ kg.

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